Multidimensional dynamics in core collapse supernovae

Jérôme Guilet (IRFU/DAp)

Crab

Cassiopeia A
Core collapse supernovae

Core collapse supernovae: type II, Ib & Ic

Not discussed here: SN Ia (thermonuclear)

Electromagnetic waves are emitted days after the explosion:

- the central engine is difficult to constrain

Gravitational waves and neutrinos (would) give a view of the instant of explosion

SN1987A: last (almost) galactic SN (LMC)
25 neutrinos detected
Observational evidence for asymmetry

Morphology of supernova remnants

Neutron star kicks: several 100 km/s
=> accelerated at birth

Polarisation of SN light:
inner ejecta are asymmetric
Core collapse: formation of a neutron star

Massive star (> 8 M$_{\odot}$)

Hydrogen

Helium

Oxygen

Iron

600 millions km

3000 km

1.4 M$_{\odot}$

Iron

Collapse of the iron core

Neutrino emission

Explosion

Stalled accretion shock

Iron

n-sphere

NS

1 sec

40 km

Jérôme Guilet (CEA Saclay) – Core collapse supernovae
Neutrino-driven mechanism: a multi-physics problem

- Multi-dimensional hydrodynamics (instabilities, turbulence..)
- General relativity
- Neutrino-matter interactions sophisticated transport schemes
- Ultra-high density equation of state
- Magnetic field
Critical neutrino luminosity

Criterion for explosion as a function of progenitor structure (Ertl et al 2015)

Two parameters:

\[ M_4 \equiv m(s = 4) \]

\[ \mu_4 \equiv \frac{dm}{dr}_{s=4} \]
Sophisticated 3D simulations are necessary

Explosion in 2D and 3D simulations? No consensus yet..

Oak ridge & japanese groups: explosions in 2D and 3D
Garching group: explosions in 2D, only for low mass in 3D with standard physics
Princeton group: first 3D explosion last month
Hydrodynamic instabilities

- Protoneutron star convection
- Neutrino-driven convection
- Standing Accretion Shock Instability (SASI)

Global asymmetry of the explosion
Consequence: faster cooling of the protoneutron star
Neutrino-driven convection: heating vs advection

Parameter controlling stability:

\[
\chi \equiv \int_{\text{gain}}^{\text{shock}} \omega_{\text{buoy}}(z) \frac{dz}{v}.
\]

\[
\chi \sim \left( \frac{G\Delta S}{H} \right)^{1/2} \frac{H}{v} \sim \left( \frac{GH}{v^2} \right)^{1/2} (\Delta S)^{1/2} \propto \text{Fr}^{-1/2}.
\]

Linear instability for \( \chi > 3 \)

For \( \chi < 3 \), convection can be non-linearly excited but not self-sustained

Kazeroni+2018

Foglizzo+2006
Neutrino-driven convection: heating vs advection

Convection helps explosion:
- Turbulence pressure pushes shock
- Increases heating efficiency
The Standing Accretion Shock Instability (SASI)

Advective-acoustic cycle

Foglizzo et al 2007

Entropy-vorticity wave

Acoustic wave

Neutron star

Shock

Purely acoustic mechanism

Blondin & Mezzacappa 2006

Acoustic wave

Neutron star

Shock

Advective-acoustic cycle favored by a WKB analysis (Foglizzo+2007, Guilet+12) & frequencies of unstable modes (Guilet+12)
SASI in models with different degree of realism

Complex comprehensive simulations
(Marek & Janka 09, Burrows et al. 06,
Wongwathanarat 10, Suwa et al. 10,
Müller et al. 12, Kuroda et al. 12,
Sumiyoshi & Yamada 12)

Multi-D hydro processes only
Blondin & Mezzacappa 07
Fernandez+2010
Kazeroni+2016,2017

SWASI experiment
Foglizzo et al. 12

progenitor structure + nuclear EOS + neutrino "transport" & interactions + "GR" + "multi-D" hydro (no magnetic field)

stationary accretion, ideal gas, 3D adiabatic

- 2D shallow water inviscid
SWASI : Shallow Water Analogue of a Shock Instability

Kitchen sink hydraulic jump

acoustic waves
shock wave
pressure

\[ \text{surface wave} \]
\[ \text{hydraulic jump} \]

\[ \text{depth} \]
SWASI: Shallow Water Analogue of a Shock Instability
Angular momentum redistribution & neutron star spins
Consequences for the spin of neutron stars

Approximate expression for the neutron star period
Guilet+14

\[ P \simeq 290 \ I_{45} \ \left( \frac{10}{\kappa} \right) \ \left( \frac{P_{\text{sasi}}}{50 \ \text{ms}} \right) \ \left( \frac{120 \ \text{km}}{r_{\text{sh}} - r_*} \right) \ \left( \frac{v_{\text{sh}}}{3000 \ \text{km.s}^{-1}} \right) \ \left( \frac{0.3 M_{\odot}.\text{s}^{-1}}{\dot{M}} \right) \ \left( \frac{150 \ \text{km}}{r_{\text{sh}}} \right)^2 \ \left( \frac{r_{\text{sh}}}{3\Delta r} \right)^2 \ \text{ms} \]

\[ \Rightarrow \text{SASI has the potential to explain the rotation of most (but not all) neutron stars} \]
Fast rotators: corotation instability
Explosion morphology revealed by nucleosynthesis

Titanium nucleosynthesis is a tracer of explosion asymmetry sensitive to electron fraction $Y_e$
Neutrino signatures

- **ν_e Burst**
  - Luminosity vs Time
  - Tests oscillation physics

- **Accretion**
  - Luminosity vs Time
  - Probes SN astrophysics

- **Cooling**
  - Luminosity vs Time
  - Probes nuclear physics & PNS convection

- Hydrodynamic instabilities

- EOS & mass dependance

Tamborra et al. 2014
Gravitational wave signature

Softer EOS
(SFHx, Steiner+13)

Stiffer EOS
(TM1, Hempel+10)

Kuroda+2016
Outstanding explosions: magnetorotational explosions?

Explosion kinetic energy:
- Typical supernova: $10^{51}$ erg
- Rare hypernova & GRB: $10^{52}$ erg
  aka type Ic BL

Total luminosity:
- Typical supernova: $10^{49}$ erg
- Superluminous supernovae: $10^{51}$ erg

Light curves can be fitted by millisecond magnetar
- strong dipole magnetic field: $B \sim 10^{14} - 10^{15}$ G
- fast rotation: $P \sim 1 - 10$ ms

e.g. Kasen+10, Dessart+12, Nicholl+13, Inserra+13
Magnetars: the most intense known magnetic fields

Magnetars
Anomalous X-ray pulsars (AXP)
Soft gamma repeater (SGR)
Strong dipole magnetic field:
\[ B \approx 10^{14} - 10^{15} \, \text{G} \]
Pulsars
\[ B \approx 10^{12} - 10^{13} \, \text{G} \]

Which supernovae are associated to magnetar birth?
A magnetar formed in NS mergers?

3 possibilities:
- direct collapse to a black hole
- hypermassive NS stabilized by rotation: delayed collapse
- stable neutron star

Formation of a magnetar?
Signature in future joint gravitational wave – electromagnetic observations?

Launch: end of 2021
GRBs: Extended emission and X-ray plateaus from magnetars?

Extraction of the magnetar rotation energy (up to $10^{53}$ erg):
- Dipole spin-down in vacuum
  
  $T_{sd} \sim 2 \times 10^3 \text{s} \left(\frac{B}{10^{15} \text{G}}\right)^{-2} \left(\frac{P}{1 \text{ms}}\right)^2$

  $L_{dip} \sim 10^{49} \text{erg/s} \left(\frac{B}{10^{15} \text{G}}\right)^2 \left(\frac{P}{1 \text{ms}}\right)^{-4} \times (1 + t/T_{sd})^{-2}$

From Gompertz+2014

Impact of a strong magnetic field on the explosion

Strong magnetic field: $B \sim 10^{15} \text{ G}$
+ fast rotation (period of few milliseconds)

$\Rightarrow$ powerful jet-driven explosions!

e.g. Sibata+06, Burrows+07, Dessart+08, Takiwaki+09,11, Winteler+12, Obergaulinger+17

But in 3D, jets may be unstable to kink instability
Moesta+2014

Caveat: origin of the magnetic field is not explained
Theoretical open question: magnetic field origin

Compression of stellar field in core collapse supernovae: $<10^{12}-10^{13}$ G ( ? )

Magnetic field of NS before merger: $10^8-10^{12}$ G

Magnetar: $10^{15}$ G

Amplification mechanism ?

Magnetorotational instability
Similar to accretion disks

Convective dynamo
Similar to planetary & stellar dynamos
The magnetorotational instability (MRI)

In ideal MHD (i.e. no resistivity or viscosity):

Condition for MRI growth: \( \frac{d\Omega}{dr} < 0 \)

Growth rate: \( \sigma = \frac{q}{2} \Omega \)

with \( \Omega \propto r^{-q} \)

→ Fast growth for fast rotation

Wavelength: \( \lambda \propto \frac{B}{\sqrt{\rho \Omega}} \)

→ Short wavelength for weak magnetic field
Proto-neutron stars vs disks conditions

MRI unstable differential rotation at radii > 10 km
Akiyama+2003, Obergaulinger+2009

Impact of conditions specific to neutron stars?
→ neutrinos
→ buoyancy (entropy & composition gradients)
→ spherical geometry

Radius (km)
Rotation profile

Ott+06

Akiyama+2003, Obergaulinger+2009

neutrinos!
Impact of neutrinos on the MRI: growth rate

Slow growth for weak initial magnetic field $< 10^{12}$ G

Comparing supernovae & neutron star mergers

=> Very similar physical conditions in NS mergers and supernovae

Guilet+2015, 2017
Numerical simulations: local models

- Small box: at a radius $r = 20$ km
  size $4 \times 4 \times 1$ km

- Differential rotation
  $\Rightarrow$ shearing periodic boundary conditions

- Entropy/composition gradients in Boussinesq approximation

Code: Snoopy (G. Lesur)

Fiducial parameters:
- $\rho = 10^{13}$ g.cm$^{-3}$
- $B = 2 \times 10^{13}$ G
- $\Omega = 2 \times 10^3$ s$^{-1}$
- $\nu = 2 \times 10^{10}$ cm$^2$.s$^{-1}$

Channel mode termination by parasitic instabilities

Rembiasz et al. 2016a&b
Impact of stratification on the MRI

Guilet & Müller (2015)

unstable buoyancy

stable stratification

color: azimuthal magnetic field

Magnetic energy units of $(10^{15} \text{ G})^2$

buoyancy parameter
Dependence on diffusion processes

\[ P_m = 10^{13} ! \]

Magnetic energy
units of \((10^{15} \text{ G})^2\)

\[ E_{\text{mag}} \propto P_m \]

\[ P_m = \text{viscosity/resistivity} \]

See also:
Fromang+2007, Lesur+2007, Meheut+2015, Potter+2017

Behaviour at realistic values: very large magnetic Prandtl number \( P_m \) ?
Global model of MRI: geometry of the magnetic field?

Simplest model of MRI in spherical geometry: incompressible, differential rotation profile forced at outer boundary


Projet de thèse d’Alexis Reboul-Salze
Global models: strength of dipole magnetic field

Equatorial dipole as strong as a magnetar 😊

$B_{\text{total}} \sim \text{a few } 10^{15} \text{ G}$

$B_{\text{dipole}} \sim \text{a few } 10^{14} \text{ G}$
First simulations of a convective dynamo in proto-neutron stars

Physics included:
- Realistic equation of state & proto-neutron star structure
- Anelastic approximation
- Only the convective zone

Is a regime with strong dipolar magnetic field possible?

Raphaël Raynaud
Magnetorotational explosions

Physics included:
- Fully compressible MHD
- Special relativity
- Neutrino transport
- Realistic equation of state
- 2D

B field amplification not described:
- test the influence of initial B field geometry & intensity

Matteo Bugli, collaboration with Martin Obergaulinger (Valencia)
Conclusions

Very rich and complex multidimensional fluid dynamics:
- PNS convection + dynamo
- Neutrino-driven convection
- Standing Accretion Shock Instability (SASI)
- Corotation instability (low-T/W)
- Magnetorotational instability (MRI)

Big impact on core collapse supernovae
- Success of explosion
- Asymmetry: neutron star kick and spin, SNR morphology
- Neutrino & gravitational wave signatures
- Magnetic fields & extreme supernovae

Multi-messenger observations will be essential to constrain all this physics

Thank you!