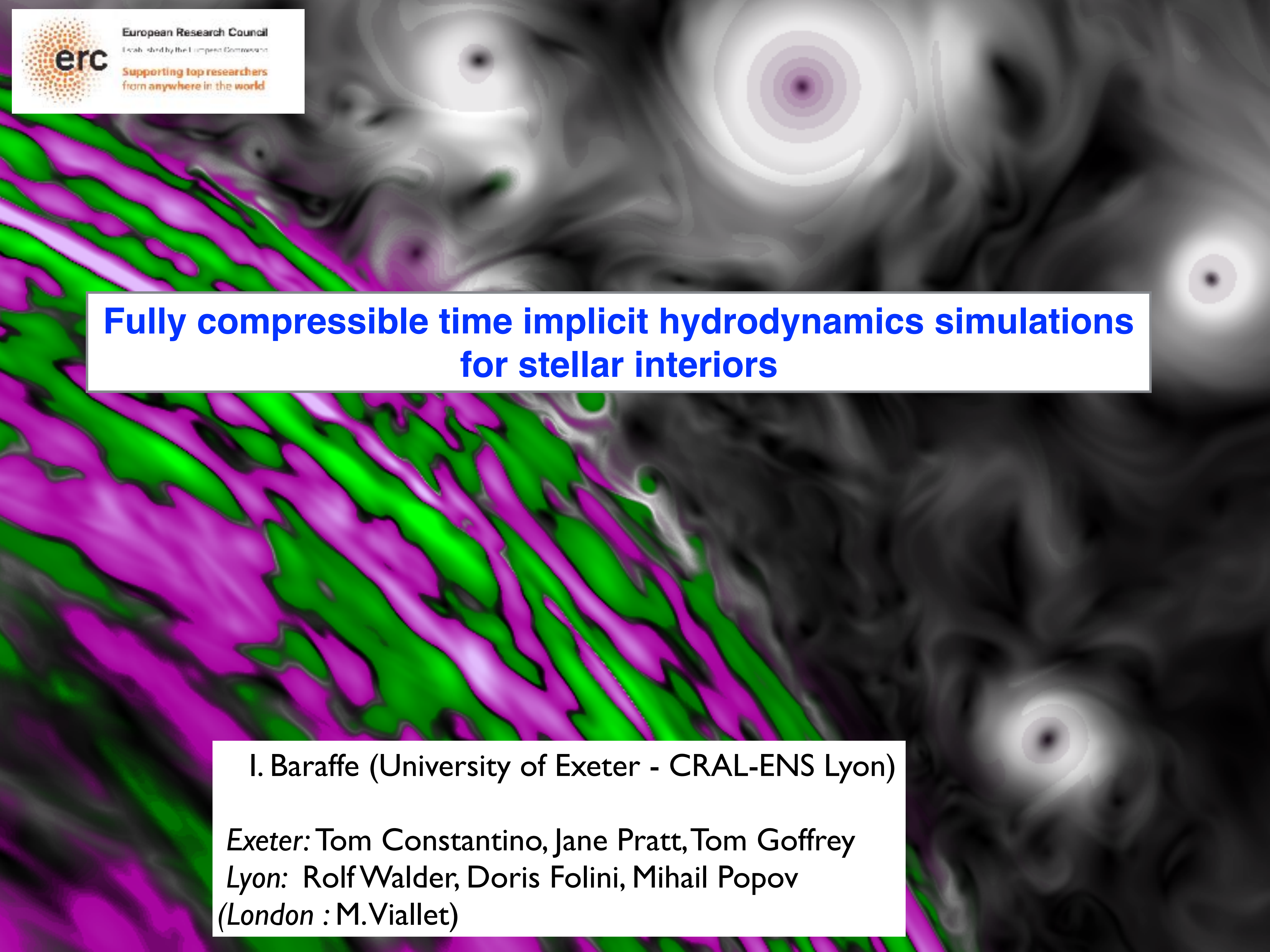


Fully compressible time implicit hydrodynamics simulations for stellar interiors

I. Baraffe (University of Exeter - CRAL-ENS Lyon)

Exeter: Tom Constantino, Jane Pratt, Tom Goffrey
Lyon: Rolf Walder, Doris Folini, Mihail Popov
(*London* : M. Viallet)

- Motivation and challenges of computing multi-dimensional stellar structures
- The fully compressible time implicit code MUSIC
- Application to convective boundary mixing (overshooting)

Motivation for multi-D time-implicit simulations in stellar physics

Characteristics of stellar interiors:

Many (M)HD processes play key roles on stellar structure and evolution

Convection, rotation, dynamo, mixing, turbulence, etc….

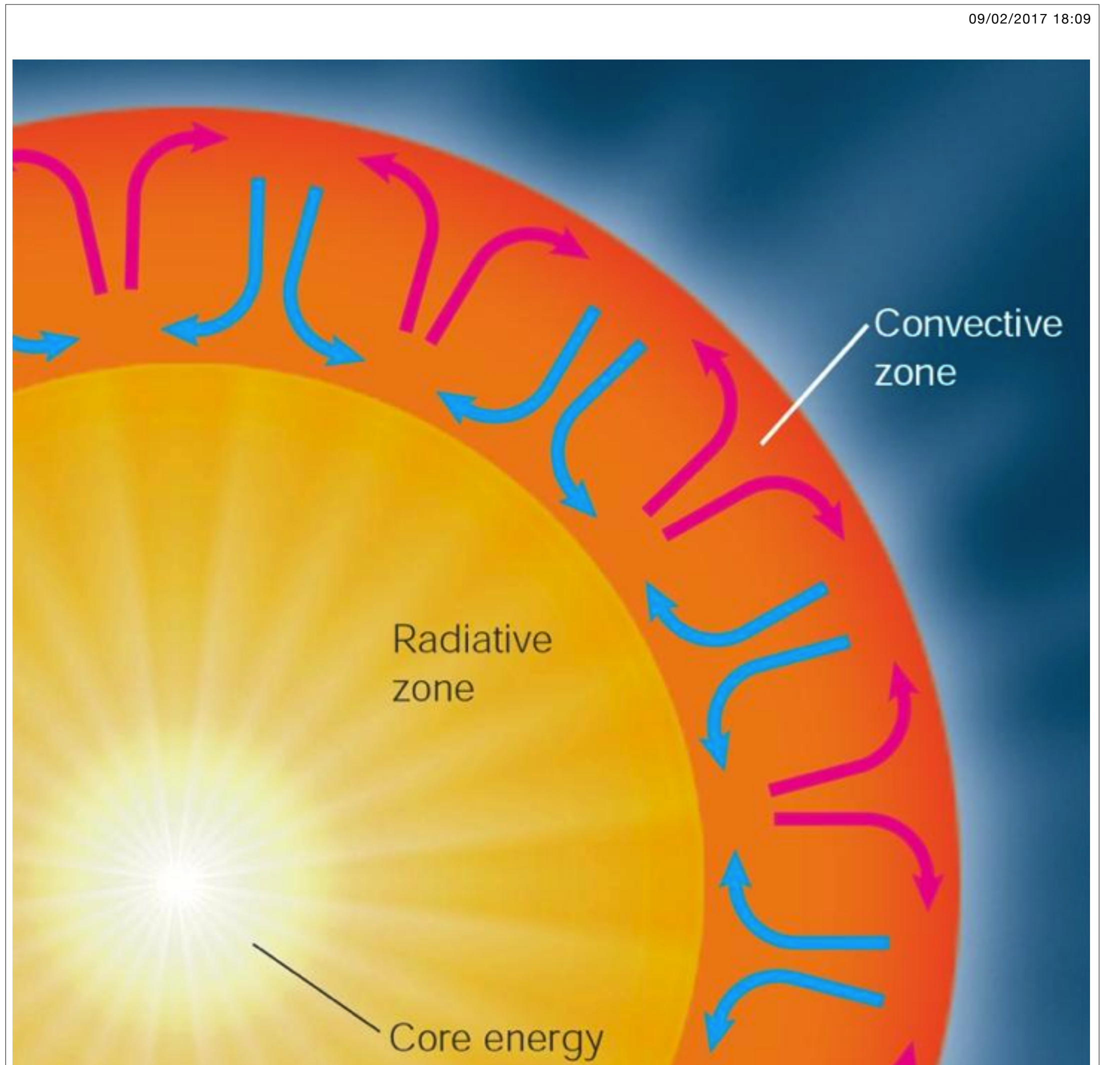
- Characterised by very different timescales

Sun $\tau_{dyn} \sim (R^3/GM)^{1/2}$ $\sim$ 30 min

τ_{conv} $\sim$ 6 days

$\tau_{thermal} \sim GM^2/(RL) \sim 2\ 10^7$ yr

τ_{nuc} $\sim 10^{10}$ yr



- Very different lengthscales

Pressure scale height: $H_P = dr/d\ln P$

centre: $H_P \sim R_{star}$ Surface: $H_P \sim 10^{-3} - 10^{-2} \times R_{star}$

- Range of Mach numbers ($M \sim 10^{-10}$ - > 1)

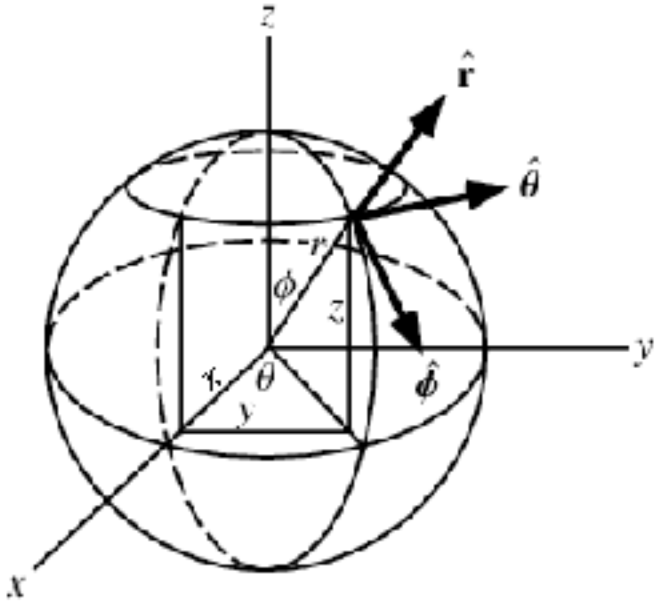
😊 Many successes of 1D (spherical symmetry) models based on phenomenological approaches—> calibration of free parameters from observations

BUT ☹️ no predictive power

☹️ degeneracy of solutions

☹️ ☹️ do we really understand the physics?

1D Phenomenological approaches have reached their limits



⇒ **Need for multi-dimensional models**

(ideally in spherical coordinates)

Development of MUSIC “Multidimensionnal Stellar Implicit Code”

(Viallet et al. 2011, 2013, 2016; Geroux et al. 2016; Pratt et al. 2016; Goffrey et al. 2016)

- Spherical geometry (2D or 3D)
- **Fully compressible** hydrodynamics

$$\frac{\partial}{\partial t}\rho = -\nabla \cdot (\rho \vec{u})$$

$$\frac{\partial}{\partial t}\rho e = -\nabla \cdot (\rho e \vec{u}) - P\nabla \cdot \vec{u} + \nabla \cdot (\chi \nabla T)$$

$$\frac{\partial}{\partial t}\rho \vec{u} = -\nabla \cdot (\rho \vec{u} \otimes \vec{u}) - \nabla P + \rho \vec{g}$$

With the radiative conductivity $\chi = 16\sigma T^3/3\kappa\rho$

κ **Rossland mean opacity** (OPAL) + **realistic equation of state** (ionisation, partial degeneracy, etc…)

- Difficulty with various disparate timescales (e.g various stiff scales)

$$\tau_{\mathrm{evol}} = \tau_{\mathrm{therm}}, \tau_{\mathrm{conv}}, \tau_{\mathrm{rot}}, \tau_{\mathrm{nuc}} \gg \tau_{\mathrm{dyn}}$$

A standard approach is to use a time-explicit integration method:

$$\frac{du(t)}{dt} = f(u(t)) \longrightarrow u^{n+1} = u^n + \Delta t f(u^n)$$

Strictly limited by the Courant- Friedrich-Lewy condition $\Delta t < \Delta t_{CFL}$

- Hydro CFL: $\Delta t_{\mathrm{hydro}} = \min \frac{\Delta x}{|u| + c_s}$ c_s speed of sound

- Radiative diffusion CFL: $\Delta t_{\mathrm{rad}} = \min \frac{\Delta x^2}{k_{\mathrm{rad}}}$ radiative diffusivity $k_{\mathrm{rad}} = \chi/\rho c_p$

For stability: $\mathrm{CFL}_{\mathrm{hydro}} = \frac{\Delta t}{\Delta t_{\mathrm{hydro}}} < 1$ **and** $\mathrm{CFL}_{\mathrm{rad}} = \frac{\Delta t}{\Delta t_{\mathrm{rad}}} < 1$

- Advective timescale: $$\Delta t_{\mathrm{adv}} = \min \frac{\Delta x}{|u|}$$

Stability limit for anelastic method $$\mathrm{CFL}_{\mathrm{adv}} = \frac{\Delta t}{\Delta t_{\mathrm{adv}}} < 1$$

- **Advantage of a time-implicit method**:

$$\frac{du(t)}{dt} = f(u(t)) \longrightarrow u^{n+1} = u^n + \Delta t f(u^{n+1})$$

No stability limit on the time-step

→ adapted for problems with various stiff scales

Time step choice is driven by accuracy and physical considerations

The equations

$$\frac{\partial}{\partial t}\rho = -\nabla\cdot(\rho\vec{u}) = R_U^{\rho}$$

$$\frac{\partial}{\partial t}\rho e = -\nabla\cdot(\rho e\vec{u}) - P\nabla\cdot\vec{u} + \nabla\cdot(\chi\nabla T) = R_U^{\rho\epsilon}$$

$$\frac{\partial}{\partial t}\rho\vec{u} = -\nabla\cdot(\rho\vec{u}\otimes\vec{u}) - \nabla P + \rho\vec{g} = R_U^{\rho\vec{u}}$$

More compactly

$$\frac{dU}{dt} = R_U(X), \quad U = (\rho, \rho\epsilon, \rho\vec{u}) \quad X = (\rho, \epsilon, \vec{u})$$

Time implicit

$$U(X^{n+1}) = U(X^n) + \frac{\Delta t}{2}\left(R_U(X^n) + R_U(X^{n+1})\right)$$

Newton-Raphson method and at each Newton iteration, solve a linear problem:

$$\underline{\mathbf{J}\delta\vec{X} = -F_U(X)}$$

- Low storage Jacobian-Free-Newton-Krylov solver (Knoll & Keyes 2004)

(Jacobian is not stored and matrix-vector products are estimated with finite-differencing)

(Viallet et al. 2016; Goffrey et al. 2017)

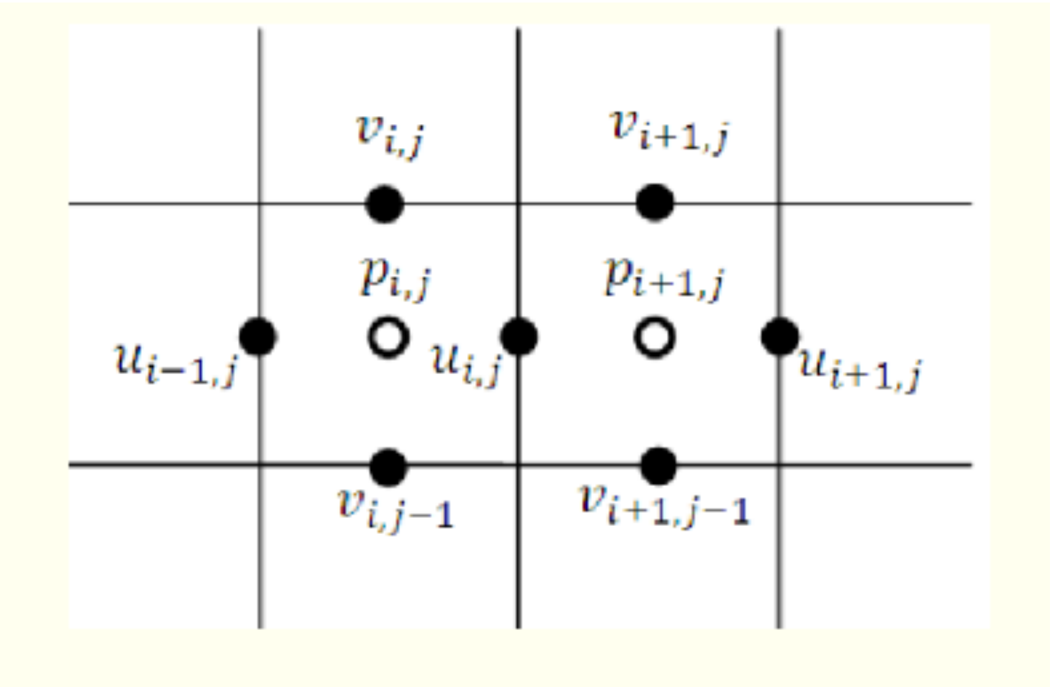
- Benchmark tests (Rayleigh-Taylor, Kelvin Helmholtz, Taylor-Green vortex)

➤ ***Accurate for a wide Mach number range M ~ 10^{-6} - 1***

(Goffrey et al. 2017)

- Finite volume method on a staggered grid

  *(really **helps for hydrostatic equilibrium*** $\nabla P = -\rho \mathbf{g}$)

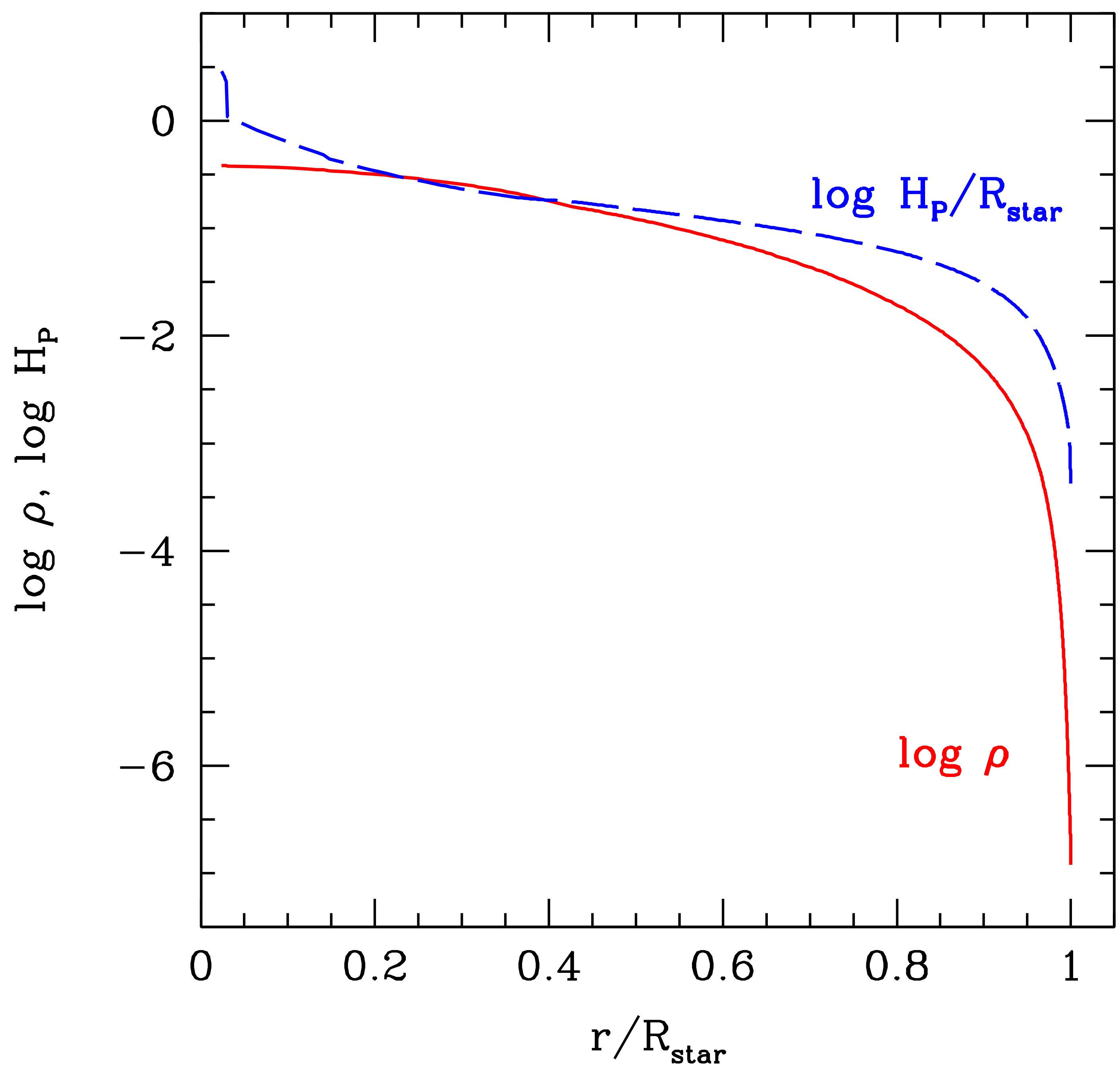


- Initial model from 1D stellar evolution calculation

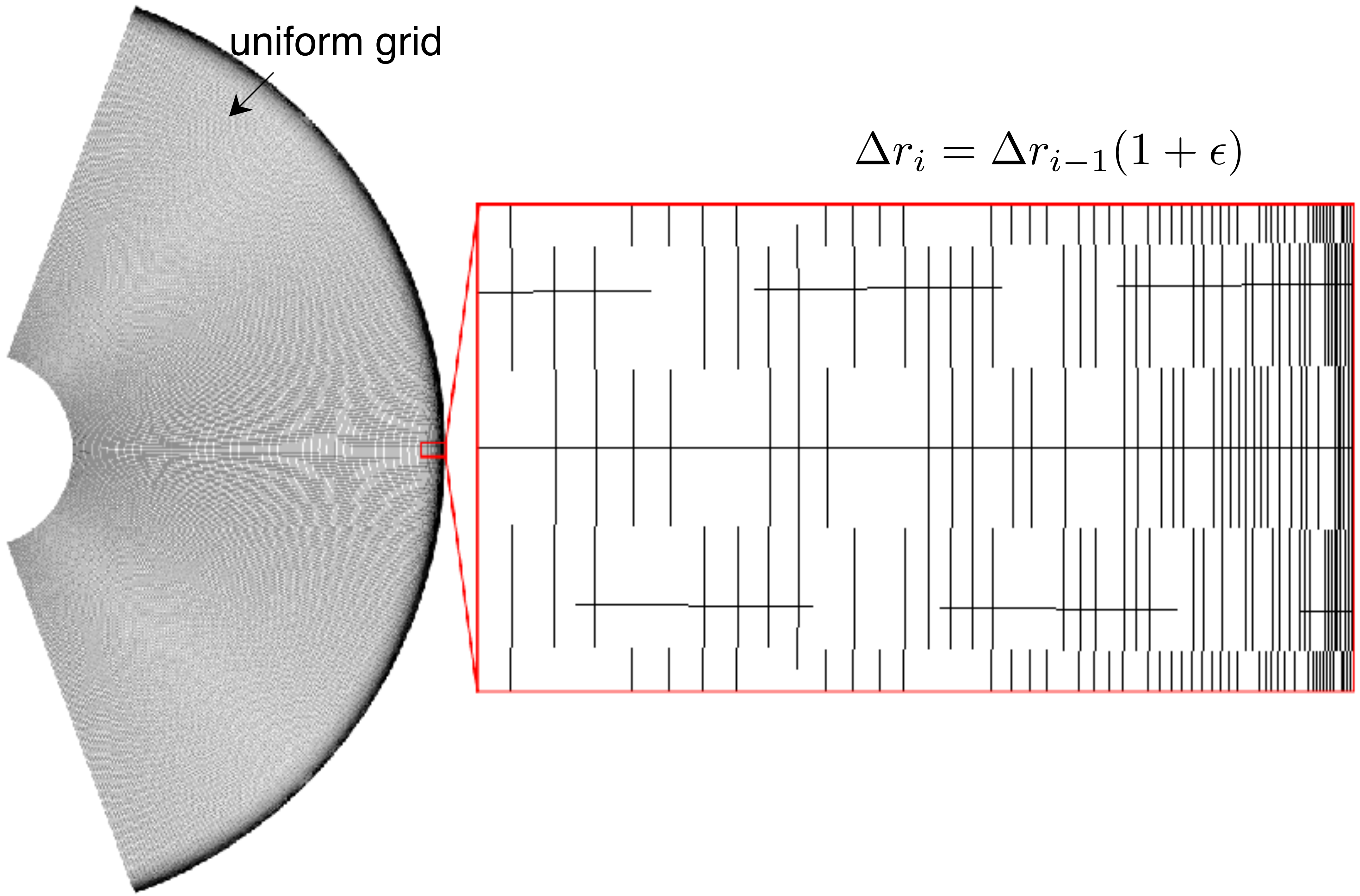
  ➤ *interface with Lyon code (Baraffe et al.) and MESA*

- **Other specificity (difficulty) characteristic of stellar interiors:**
  - Very different spatial scales from the centre to the surface:
    pressure scale height H_P varies by several orders of magnitude

  - Very steep gradients close to the surface



⇒ Use of a non-uniform grid at surface to resolve smaller scales/steep gradients

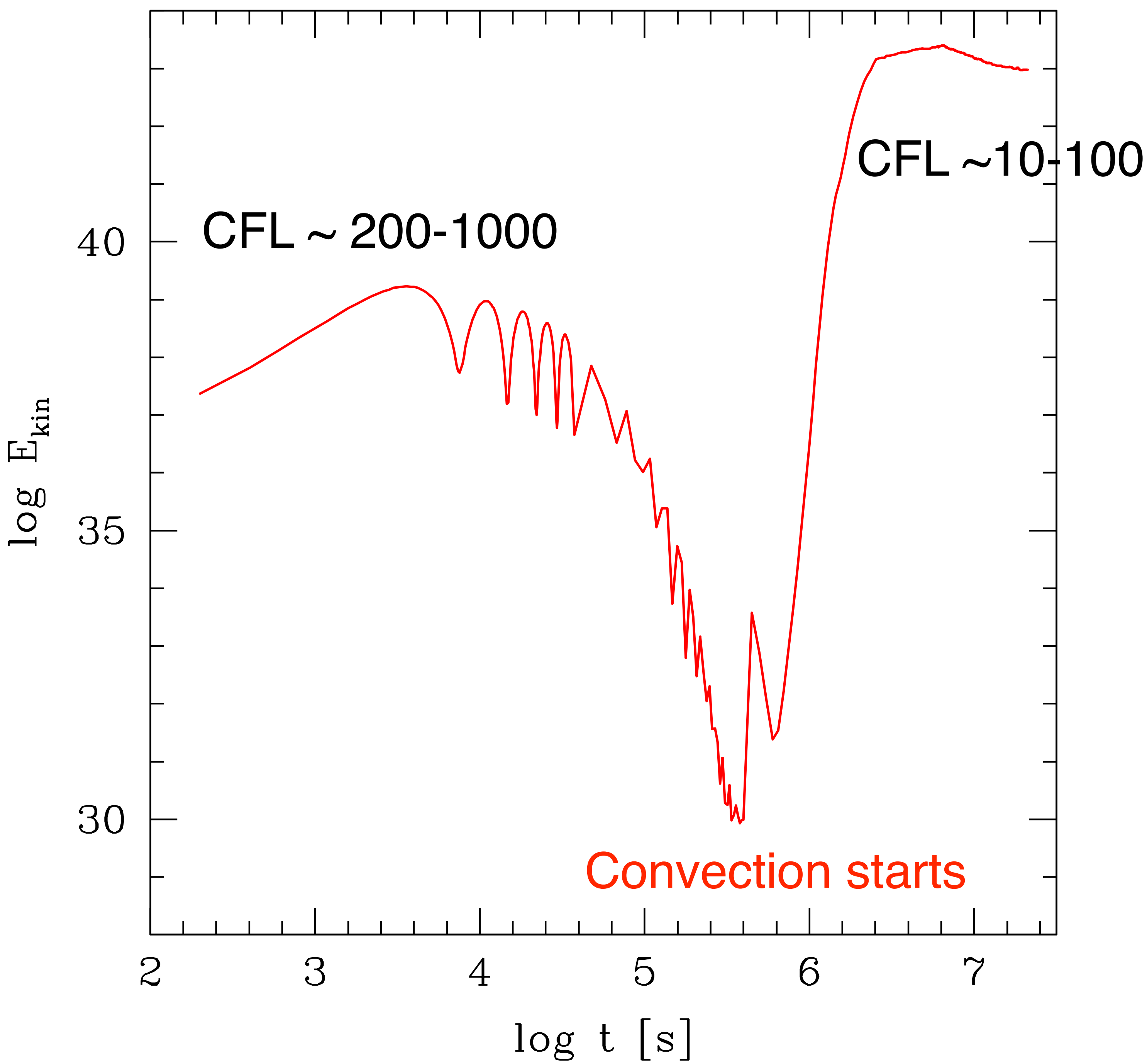


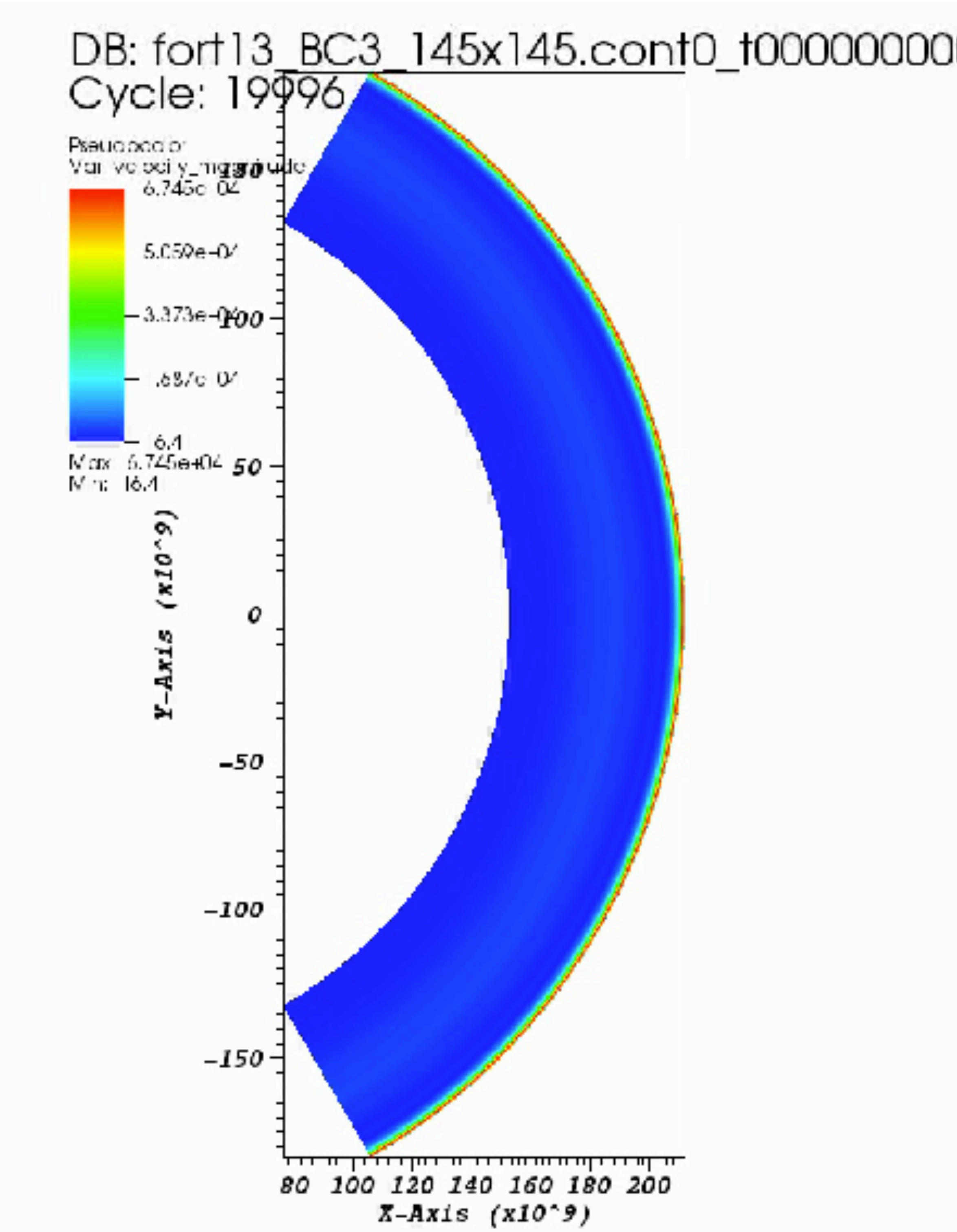
$$\Delta r_i = \Delta r_{i-1}(1+\epsilon)$$

Advantage of time implicit solver —> speed up the relaxation phase starting from 1D initial model (large time steps + stability)

*(**key** to explore a range of parameters like stellar masses)*

Relaxation phase from a 1D initial stellar model





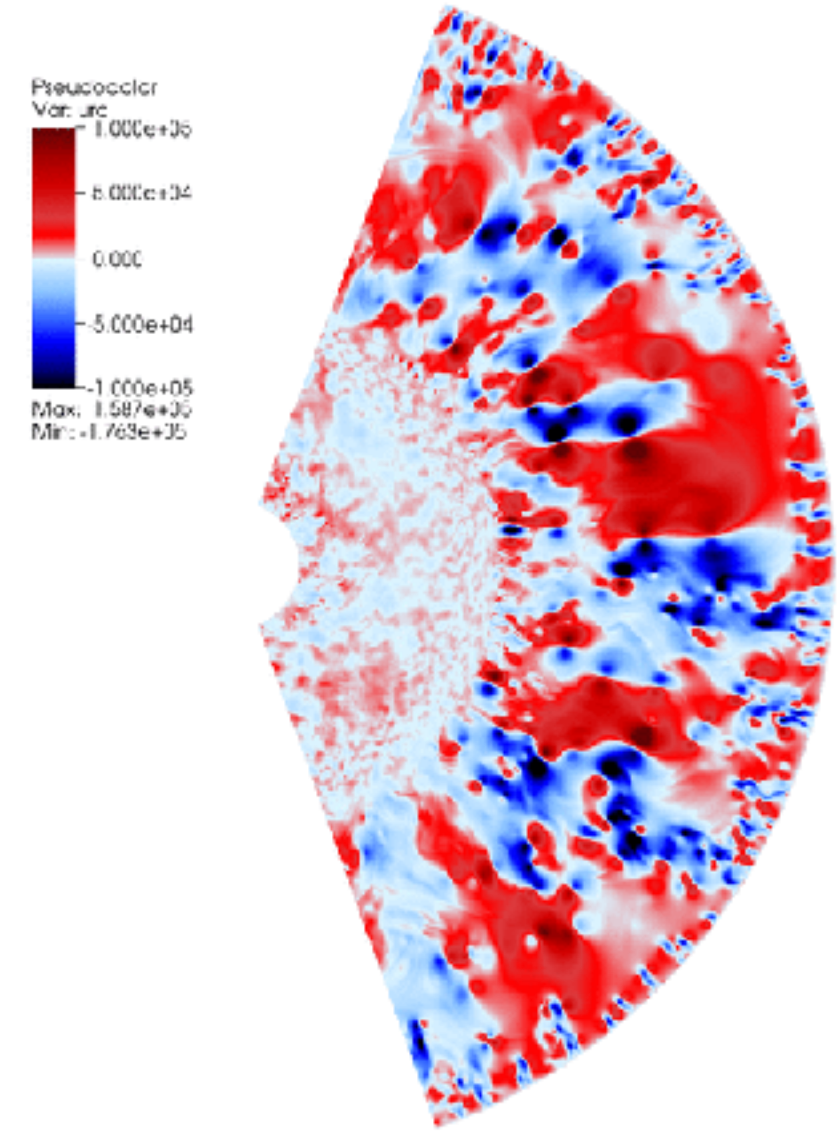
$CFL = \Delta t / \Delta t_{CFL}$ $\quad \Delta t_{CFL} = \min [\Delta x / (u + c_{sound})]$

Performance of MUSIC:
Simulations of a star in 2D/3D slices from central region to surface

Example for a young (pre main-sequence) star (1 $M_{\odot}$, ~ 60% convective envelope)

- 2D simulations 256^2 (r/R= 0.2 - 0.94)
  ~ 10 convective turnover timescales (τ_{conv} ~ *10^6 s) —> ~ 5hr wallclock time with 16 procs*

  Comparison with time explicit code (A-MAZE) —> ~ 2 weeks wallclock time



- 2D simulations 1024^2 (r/R = 0.1 - 1)

  256 procs, 72hr wallclock time for one τ_{conv}

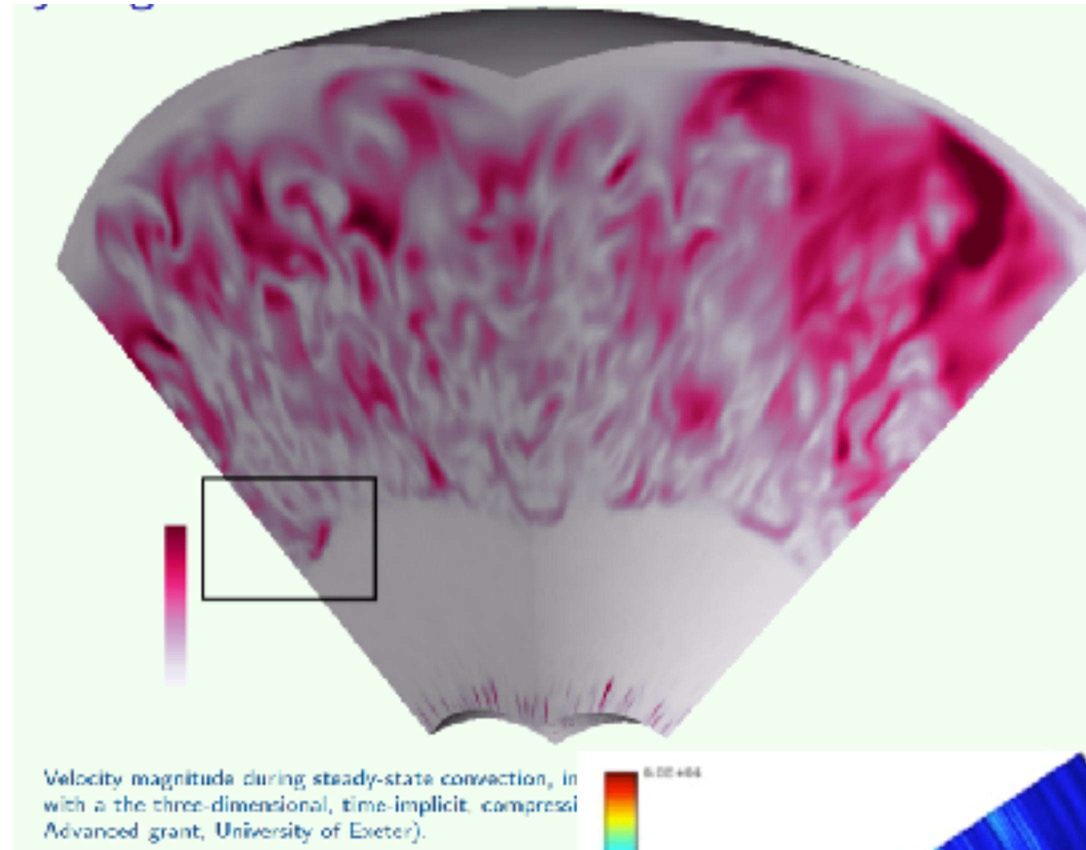
- 3D simulations 256^3
  with *512 procs, 6 days wallclock time for one τ_{conv}*

Applications to the problem of convective boundary mixing (overshooting/penetration) in stars

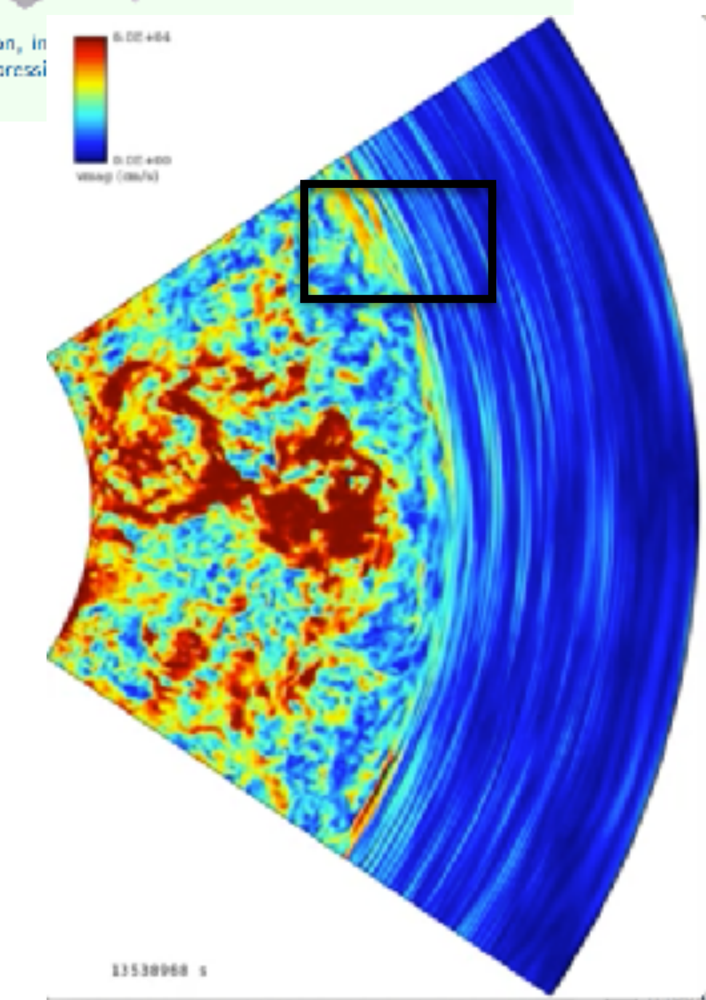
Long standing problem affecting chemical mixing, **age of stars**, transport of angular momentum and magnetic field.

☛ Great constraints from asteroseismology

(Roxburgh 1965; Shaviv & Salpeter 1973; Schmitt et al 1984, etc…)



Envelope

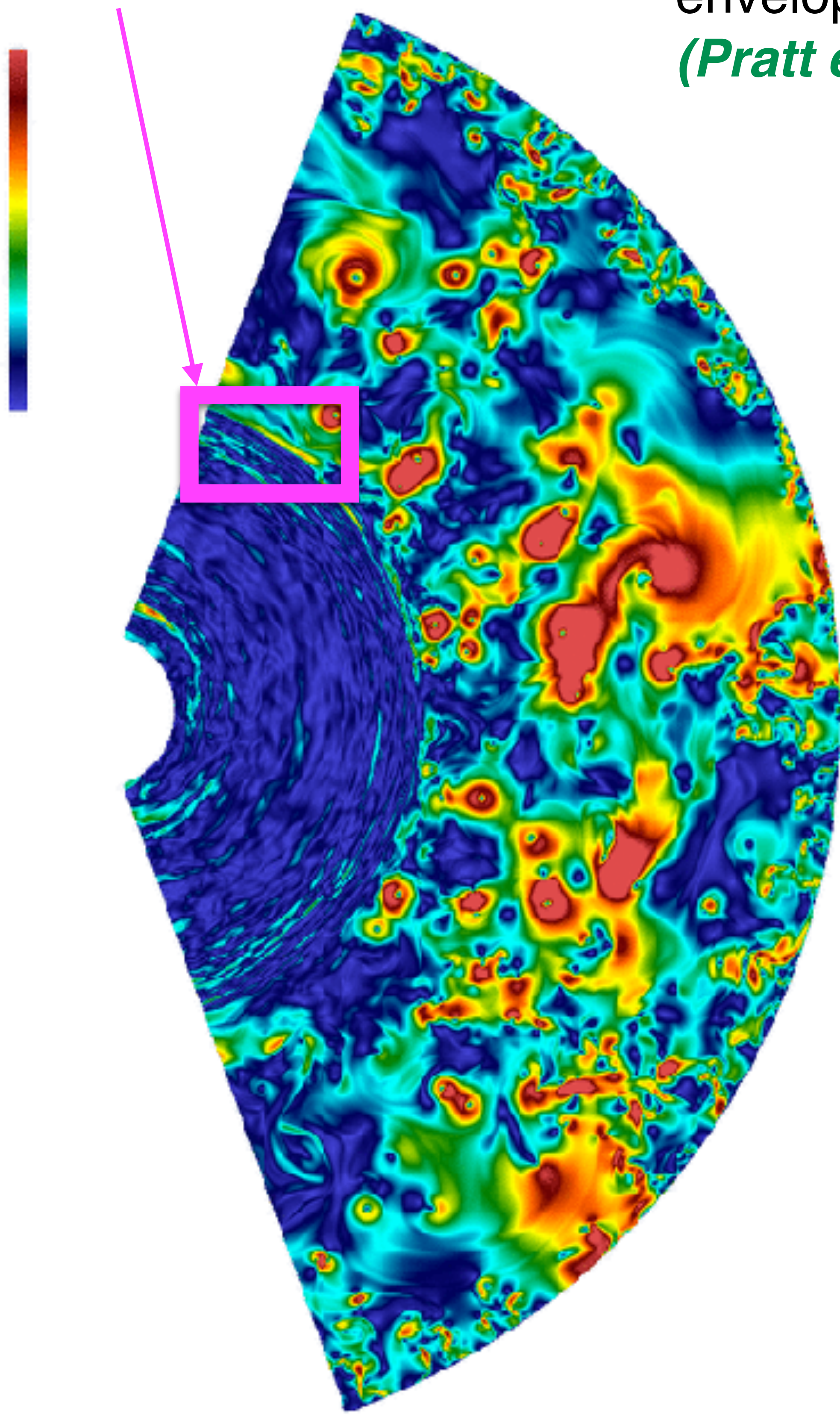


Core

Standard treatment in 1D codes: mixing over an arbitrary width $d_{ov} = \alpha H_P$ (α free parameter)

☛Application to convective boundary mixing in stellar envelopes

Penetration region
Convective boundary mixing



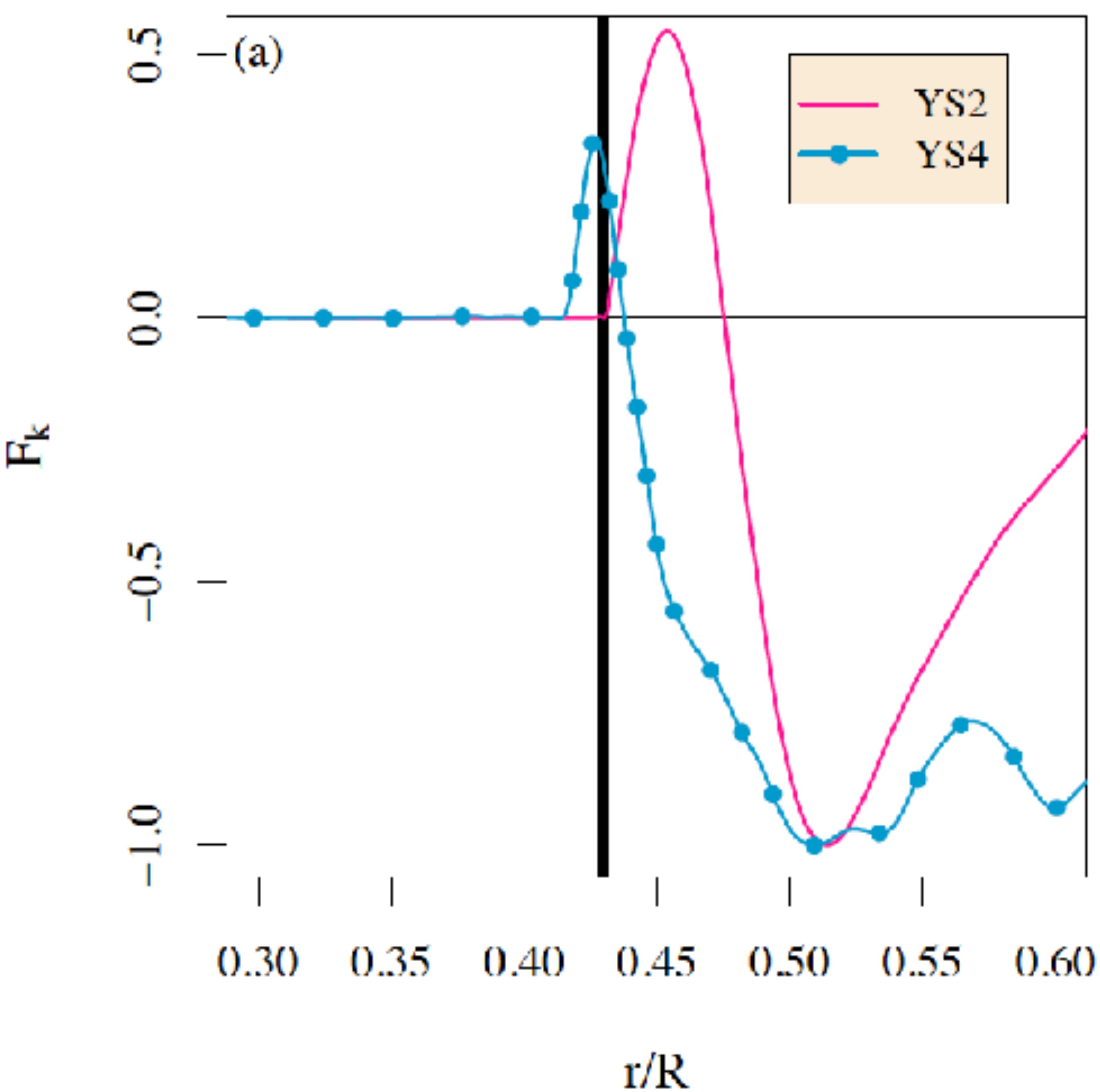
Analysis of 2D/3D simulations of a star with a large convective envelope and a radiative core (Pre-main sequence star) *(Pratt et al. 2016, 2017)*

☛ Goal: Derivation of a **diffusion coefficient** D(r) characterising the mixing in the transition region

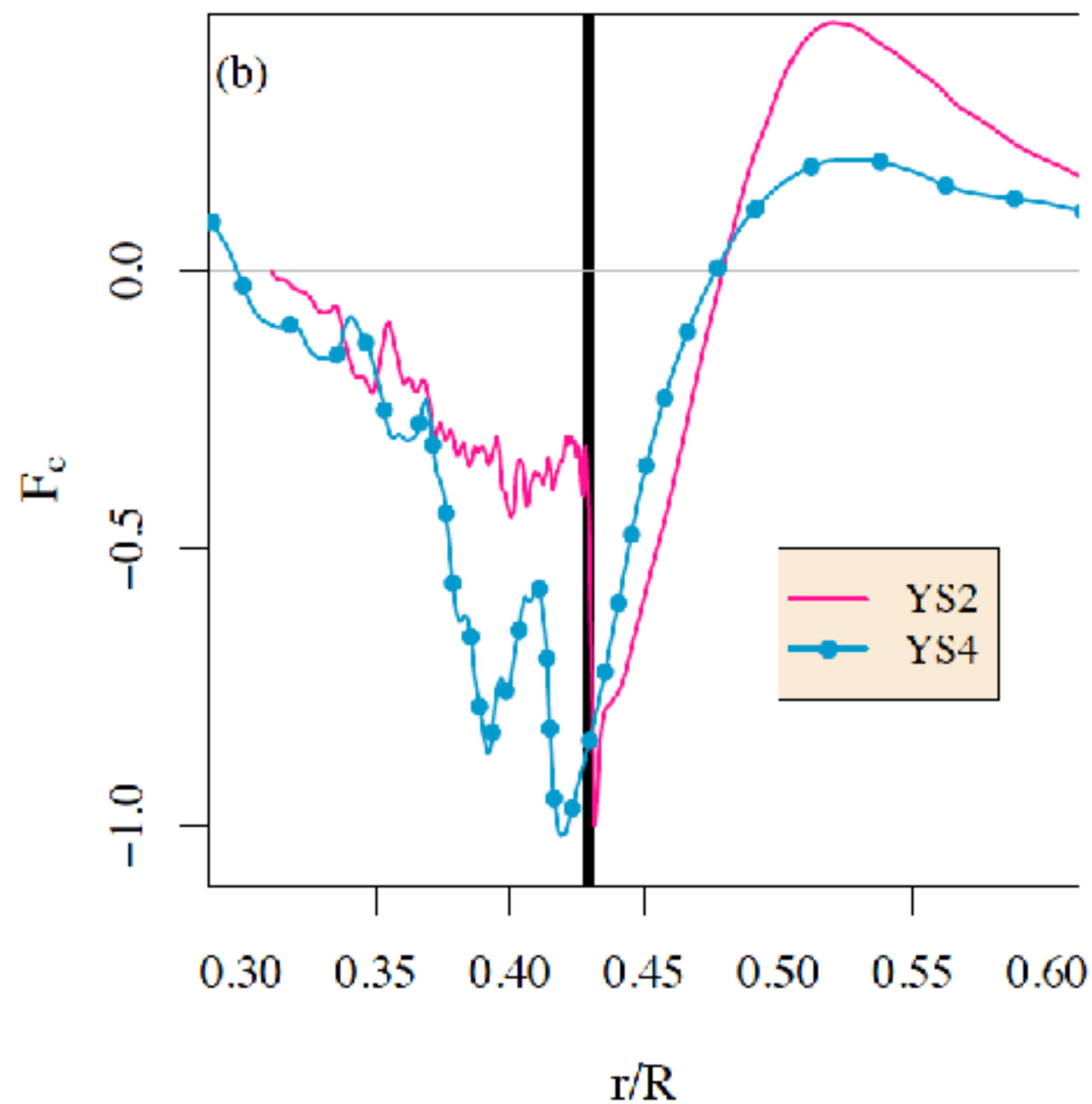
Velocity magnitude : very high res 2432x2048

Overshooting region characterised by positive vertical kinetic energy flux and/or by change of sign of the vertical heat flux (e.g Hurlburt et al. 1994; Rogers et al. 2006)

$F_k = 1/2 \, v_r \, \rho \, v^2$



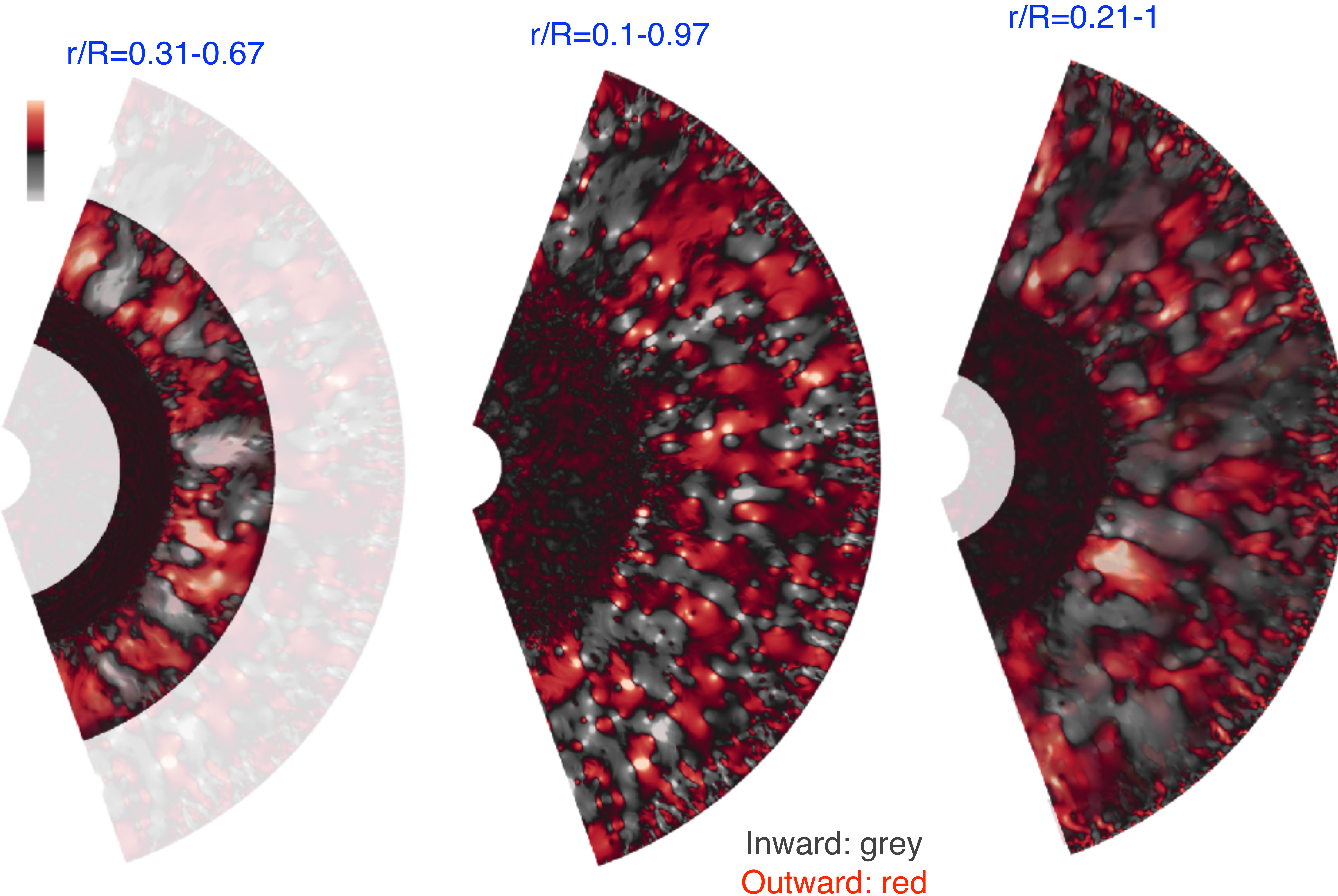
$F_{heat} = \rho \, v_r \, (\delta T) \, c_P$



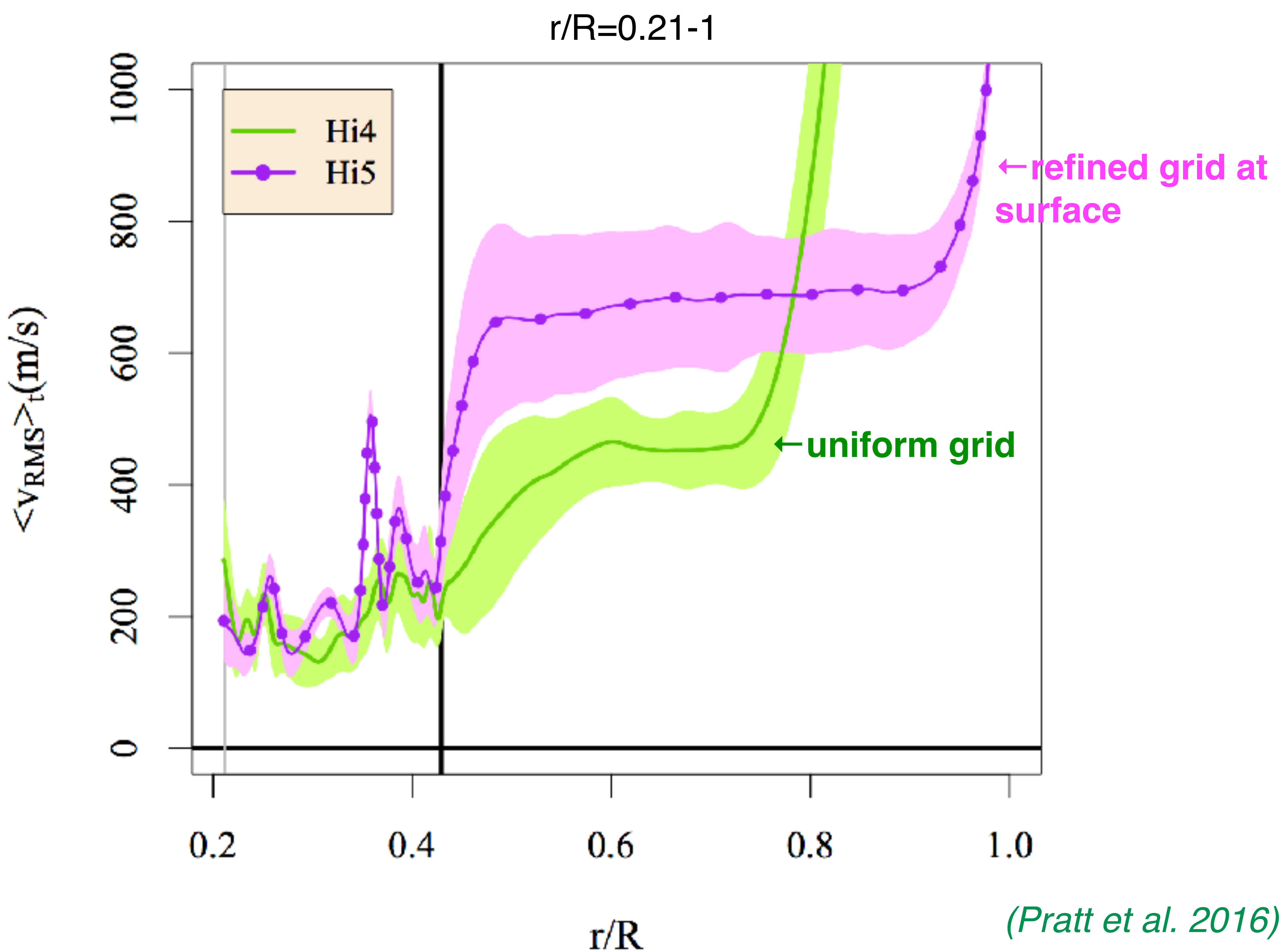
Time-averaged and horizontally volume-averaged fluxes (usual procedure)

Non locality of convection: effect of the boundaries on the structures and velocities

(Pratt et al. 2016; 2017)

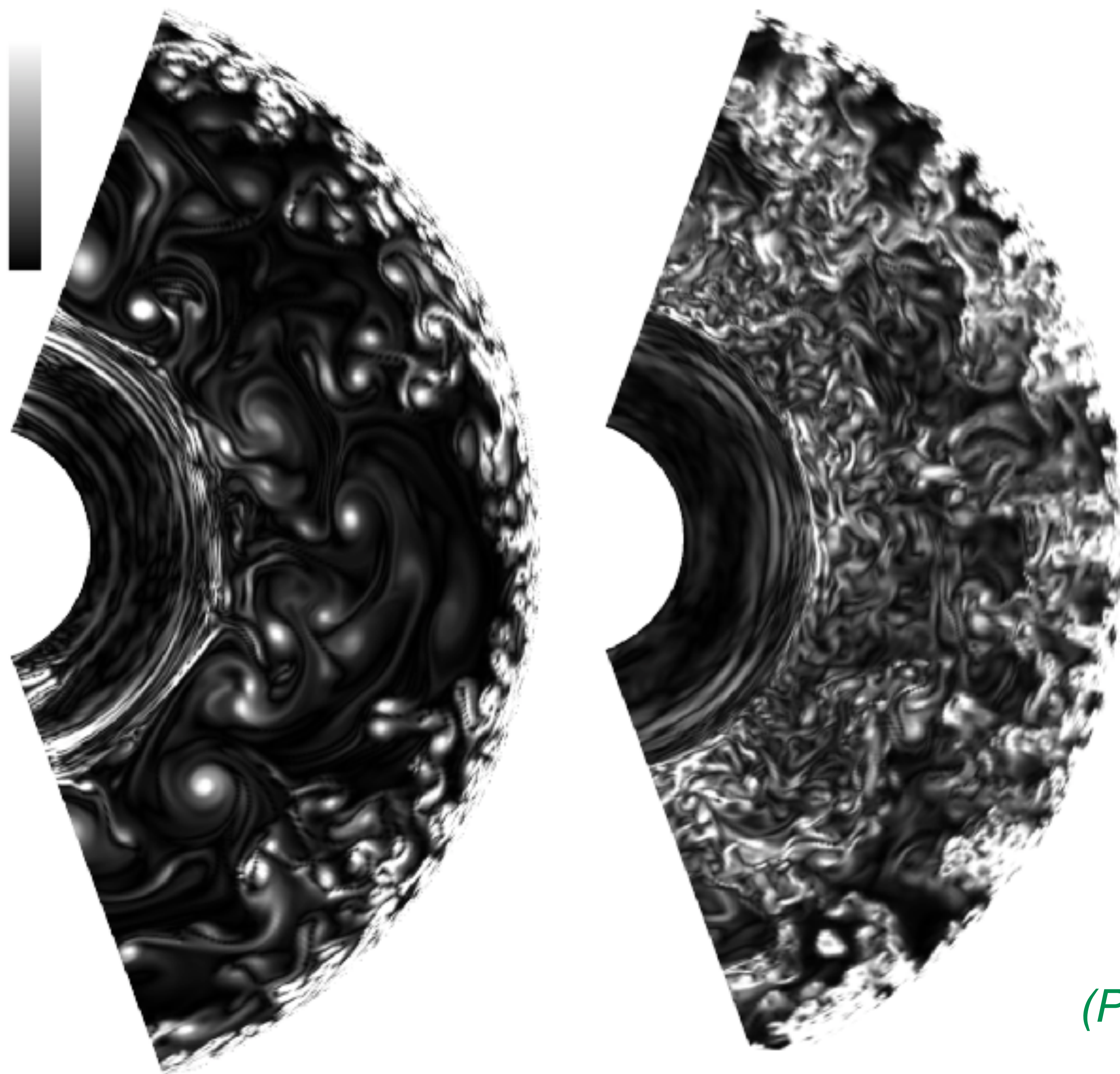


Impact of the surface treatment on the velocities in the convective zone and at the interface convection/radiation



(Pratt et al. 2016)

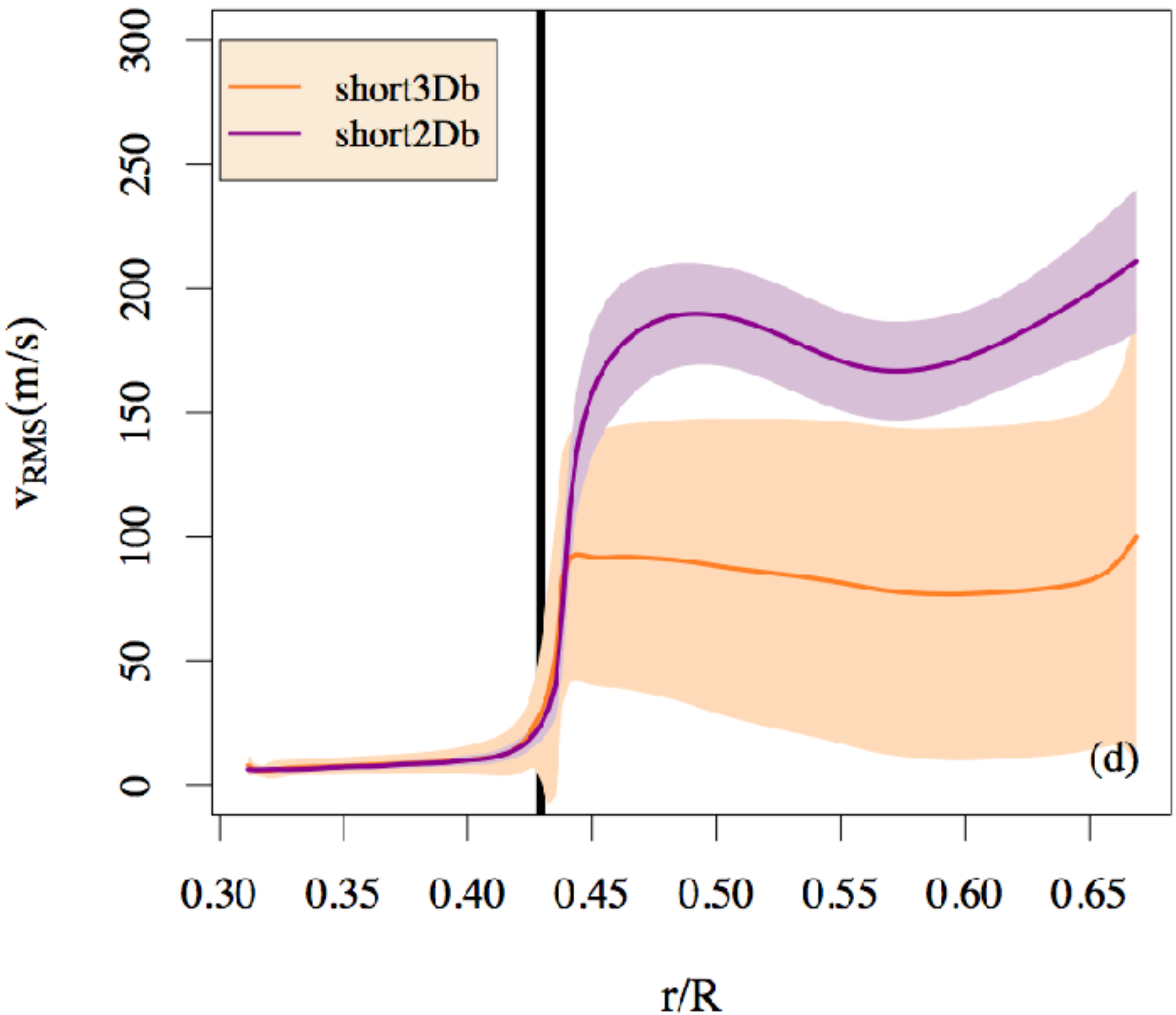
Differences between 2D and 3D simulations: vorticity magnitude



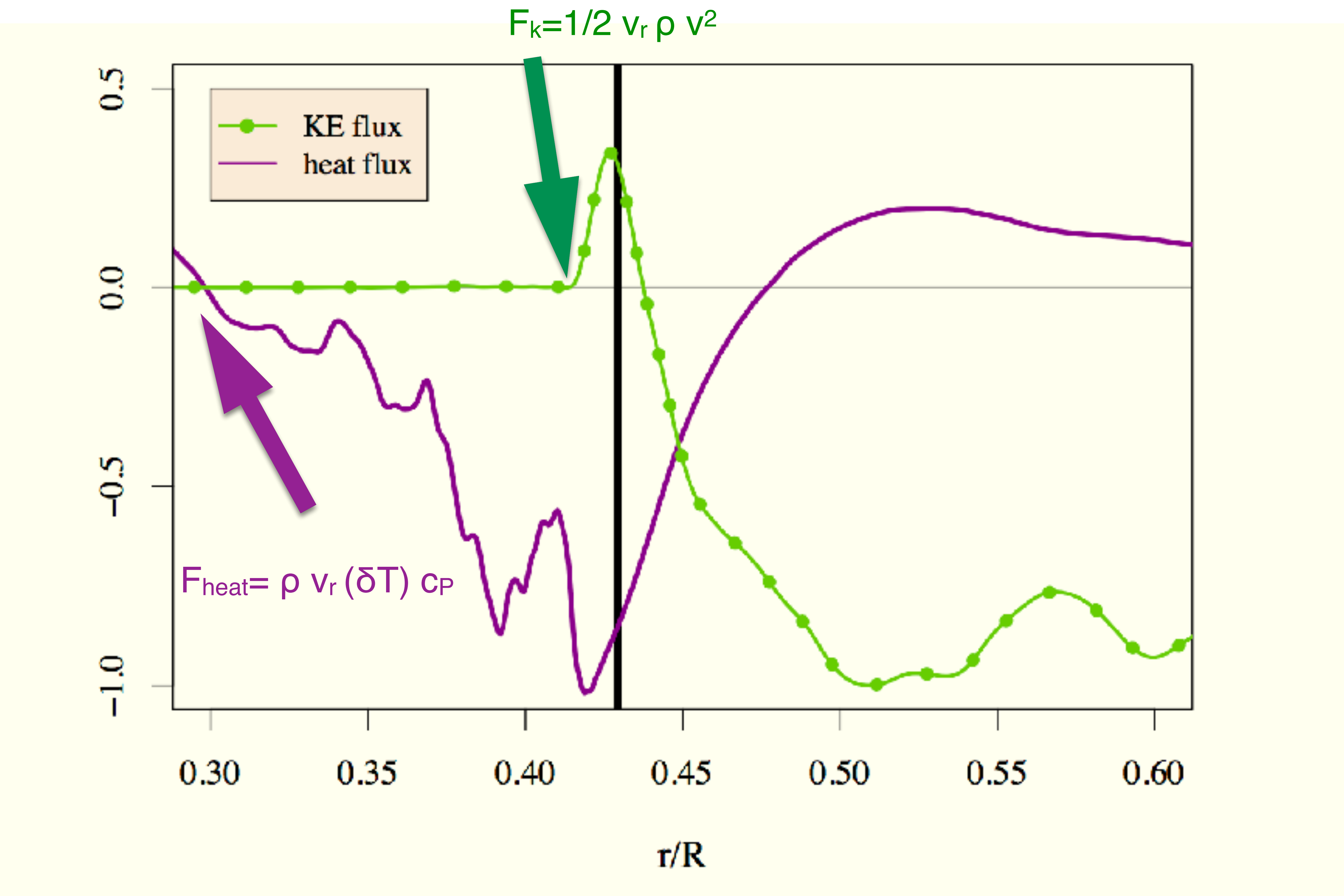
(Pratt et al. 2018)

Fig. 5. Typical snapshot of the vorticity magnitude in simulation wide2D (upper left), and in a two-dimensional cut of simulation wide3D (upper right). Color scales are identical.

Differences between 2D and 3D simulations: velocity magnitude

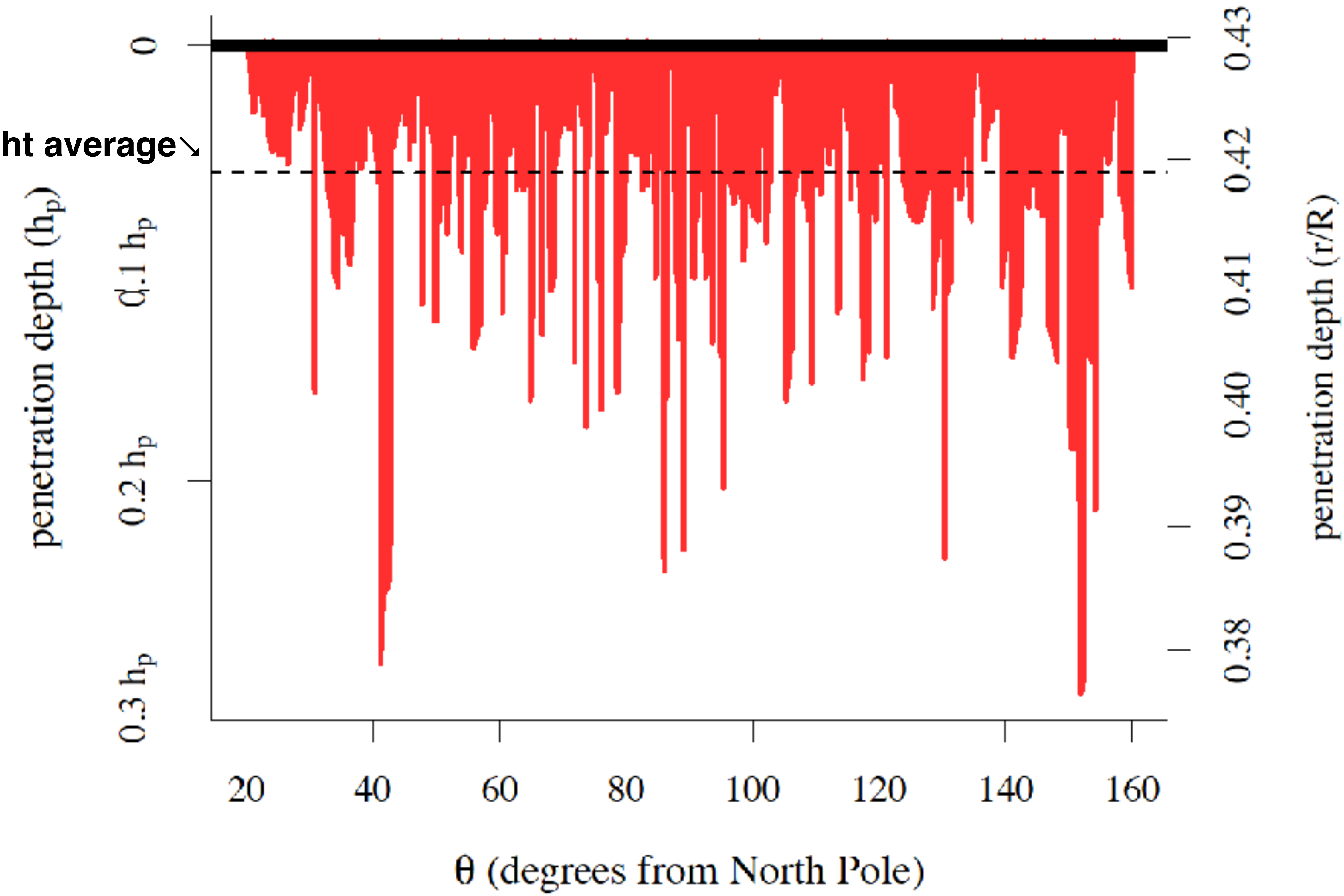


Horizontally and time-averaged F_{kin} and F_{heat} —> give different overshooting width



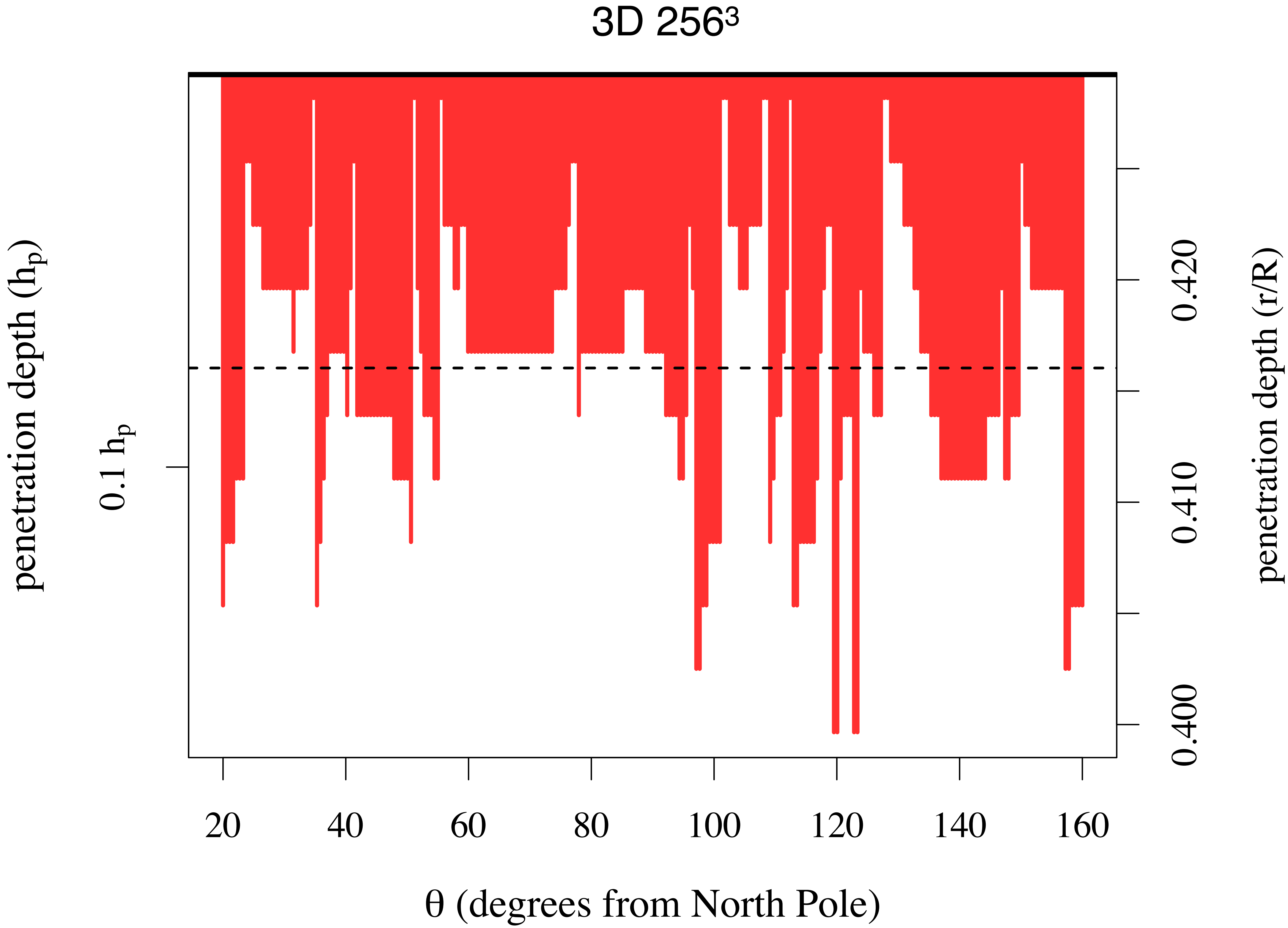
Typical shape of the penetration depths (at a given time): extent of downflows beyond the convective boundary varies with colatitude θ

straight average↘



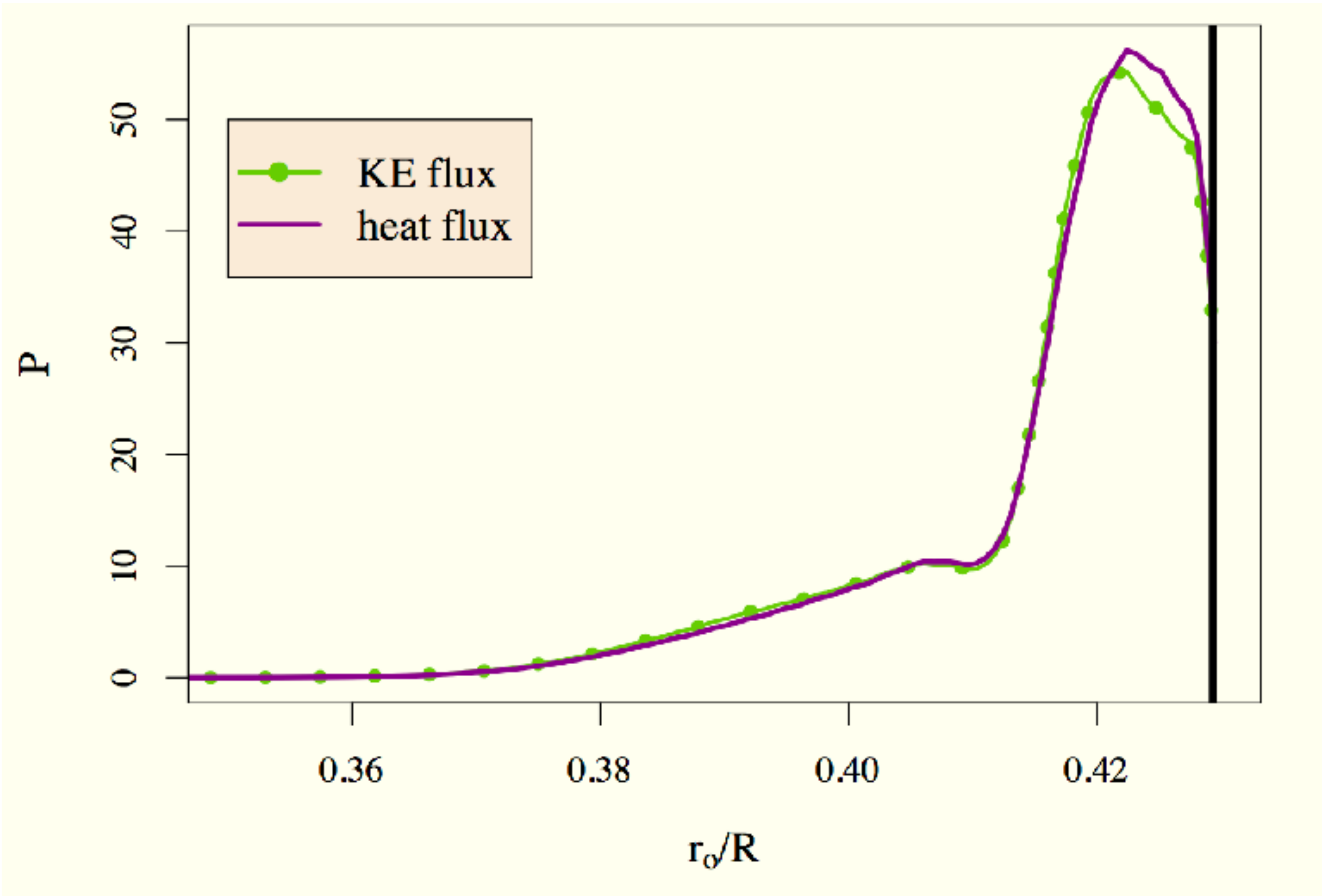
☛ **Straight average miss the larger penetration events**

Note: Qualitatively the same picture with 3D simulations



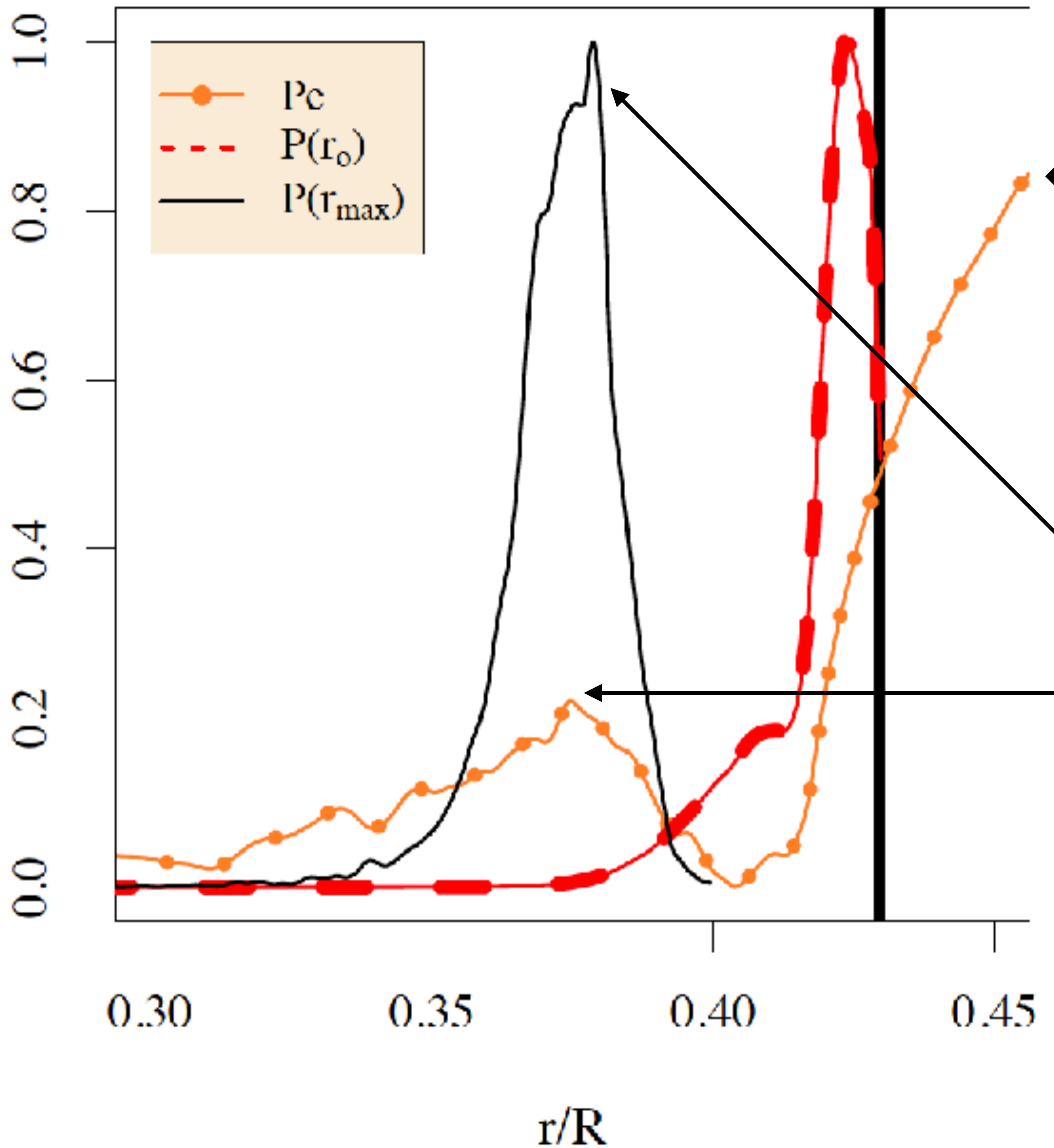
Probability Distribution of penetration depths r_0 (depth of downflows penetrating at all angles and sampled at fixed time intervals $\Delta t = 10^3$ s ~ 1/1000 τ_{conv})

PDF (KE) and PDF(heat) for the same data are remarkably similar



(τ_{conv} ~ 3 10^6 s (τ_{dyn} ~ 4 hr) - up to 500 τ_{conv})

Comparison between the probability distribution (PD) of penetration depths r_0 for all downflows and the PD of maximal penetration depth r_{max} at any time

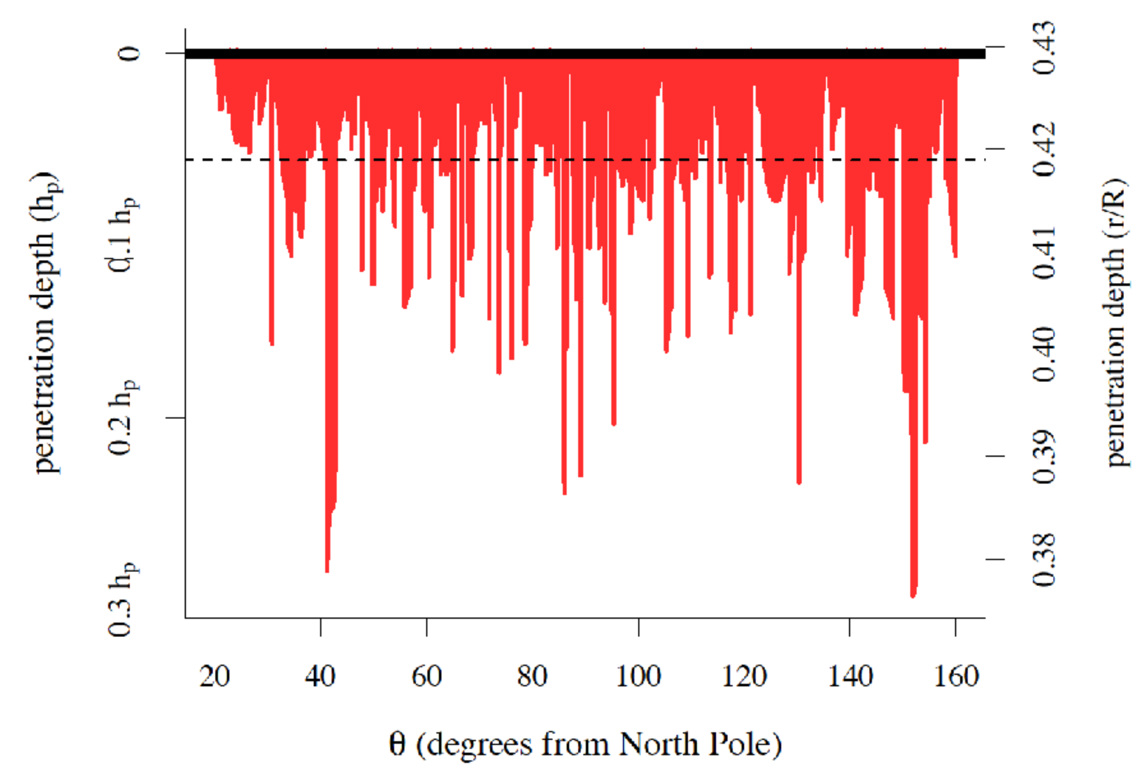


Peclet number

← $Pe = v_{rms} H_p/K_T$

($K_T = \chi/\rho c_P$ thermal diffusivity)

(measure of advective versus diffusive transport)



Define a maximum penetration length $\Delta r_{max}(t) = \max | r_0(t) - r_{conv} |$ (over all angles at a given time t)

☛ Distribution of maximal penetration depths, linked to extreme events in the tail of the distribution, can be described by **extreme value theory**

☛ Determine the probability of events that are more extreme than any previously observed (used in Earth science, traffic prediction, unusually large flooding event, finance…)

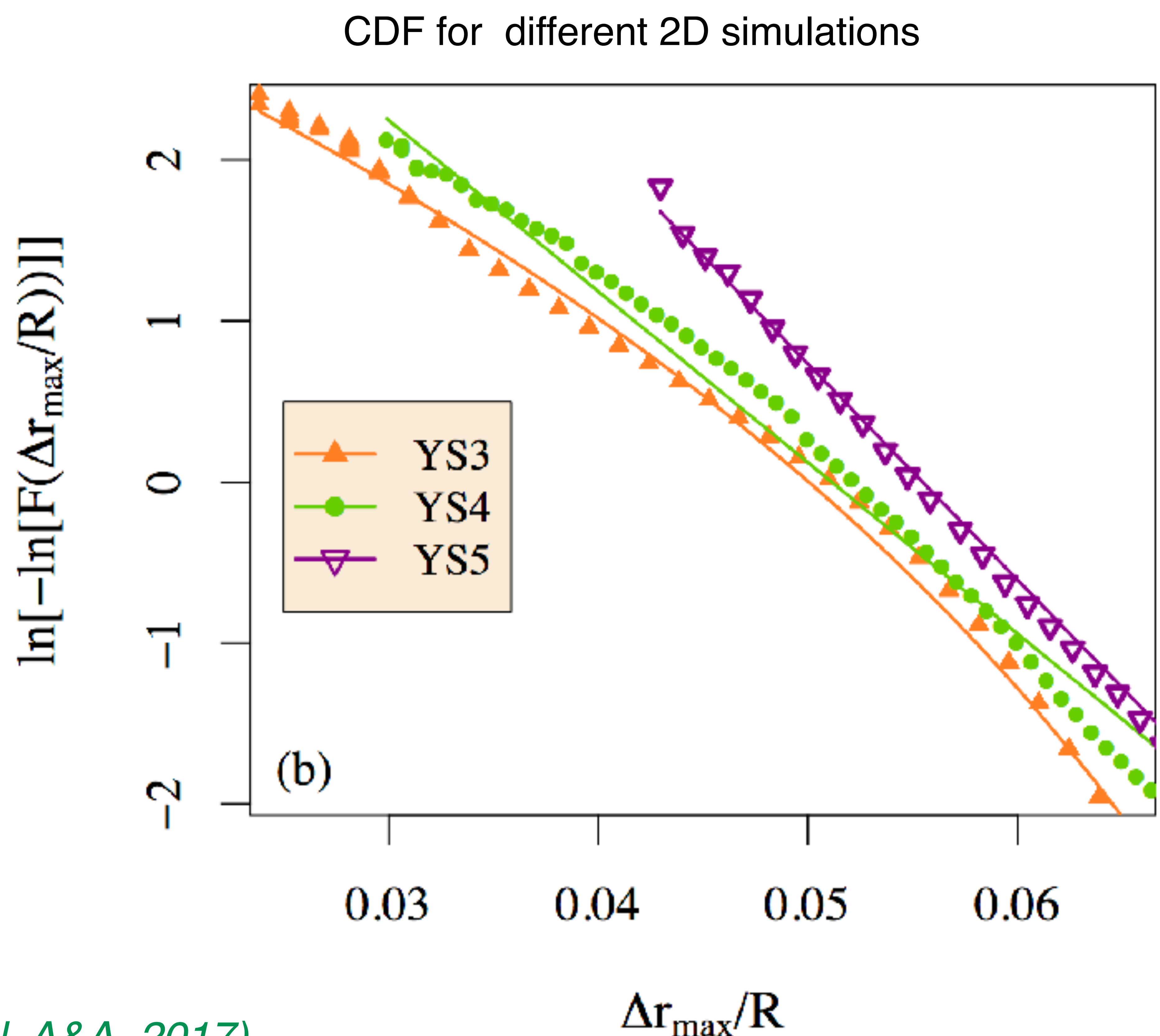
- Cumulative distribution function based on the extreme value distribution function to model the probability of maximal events has the general form:

$$F(x) = \exp\left(-\left(1 + \kappa\left(\frac{x-\mu}{\lambda}\right)\right)^{-1/\kappa}\right)$$

Where k is defined as the shape parameter. If k=0, the form reduces to:

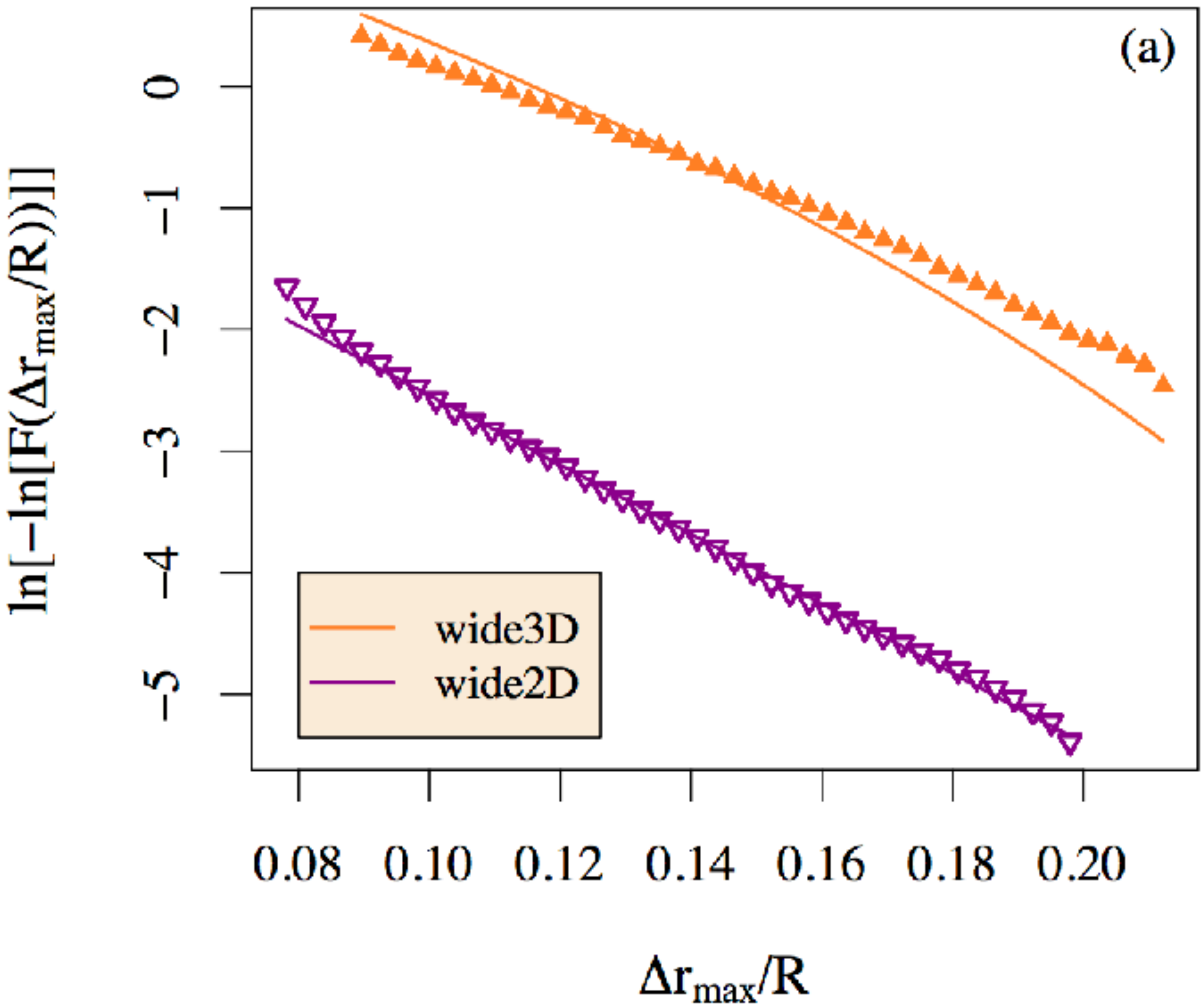
$$F(x) = \exp\left[-\exp\left(-\left(\frac{x-\mu}{\lambda}\right)\right)\right].$$

For most simulations, **best fit** given by a CDF $\propto$ exp(-exp(f(x))) form (with k —> 0).



(Pratt et al. A&A, 2017)

CDF for 2D and 3D simulations

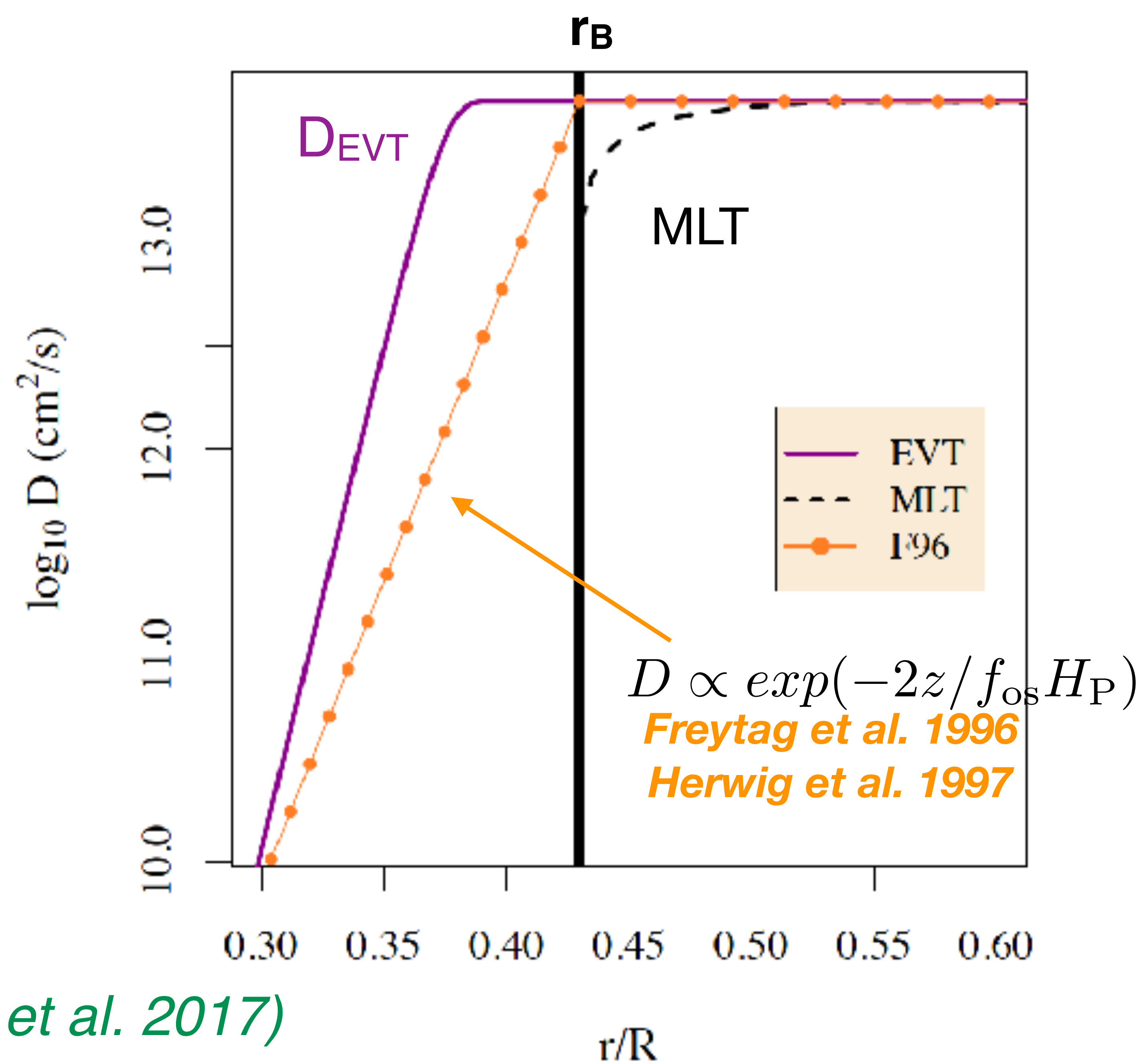


(Pratt et al. A&A, 2018)

☛ Our idea is to associate a **diffusion coefficient** to the CDF for maximum penetration events

$$D_{EVT} = D_0 \left\{ 1 - \exp\left[- \exp\left(- \frac{(r_{\rm B} - r) - \mu}{\lambda} \right) \right] \right\}$$

λ, μ are predicted by simulation



(Pratt et al. A&A, 2017; Baraffe et al. 2017)

Conclusion ☛ Next steps

- **Envelope overshooting**
  - Extension of multi-D simulations to **different masses/ages and rotation rates**
  - Explore **MHD effects**
  - Analyse the **heat transport** in the overshooting region

- Test new convective boundary mixing formalisms against observational constraints:
  - Li as a function of age and rotation in solar type stars
  - Speed of sound profile in the transition region of the Sun

- **Extend our 2D/3D simulations to convective core overshooting:**
  - Can we apply the same statistical approach i.e presence of extreme penetrating plumes responsible for the mixing?
  - Test new transport coefficients (chemical species, heat) against asteroseismology

    Search for signatures on mode properties that can be diagnosed by asteroseimology

- **Exploit the idea of statistical methods by exploring other rare even algorithms**