The pseudo-spectral code MagIC

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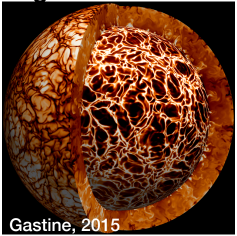
MagIC overview

What for ?

- fluid dynamics a in a rotating spherical shell
- solve for the coupled evolution of
  1. sound-proof approximations of the Navier-Stokes equation
  2. heat transfer equation
  3. chemical composition equation
  4. induction equation
- mixed implicit/explicit time step scheme
- hybrid OpenMP/MPI parallelisation,
  scales up to 1000 processors

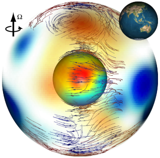
Applications: from geophysics to stellar physics

Rayleigh-Bénard convection



Gastine, 2015

Geodynamo models

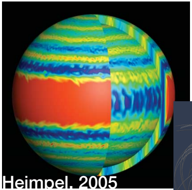


Aubert, 2008

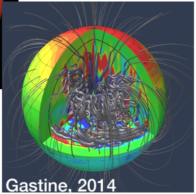
Spherical Couette dynamo

Cao, 2012

Jupiter: zonal jets, dynamo



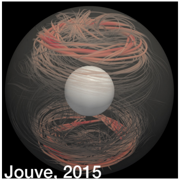
Heimpel, 2005



Gastine, 2014

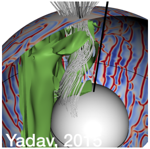
Stellar physics

MRI in a spherical shell



Jouve, 2015

Formation of polar spots in fully convective stars



Yadav, 2015

How ?

- solenoidal fields $\Longrightarrow$ poloidal/toroidal decomposition

$$\nabla \cdot \mathbf{B} = 0 \iff \mathbf{B} = \boldsymbol{\nabla} \times \boldsymbol{\nabla} \times B_P \mathbf{e}_r + \boldsymbol{\nabla} \times B_T \mathbf{e}_r$$

- spherical harmonic decomposition in the angular directions

$$F(r,\theta,\varphi) = \sum_{l=0}^{+\infty} \sum_{m=0}^{l} f_l^m(r) Y_l^m(\theta,\varphi) + \text{c. c.}$$

$$\Delta_H Y_l^m = l(l+1)/r^2 Y_l^m$$

- Chebyshev polynomials/finite differences in the radial direction
- pseudo-spectral : non-linear terms treated in the physical space (gain in computational speed)

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Dimensionless formulation

Reference units : BE CAREFUL !

$$[d] = r_o - r_i \quad \text{(shell width)}$$

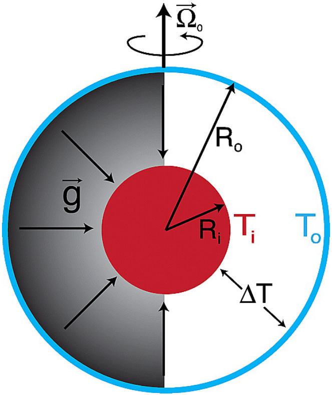
$$[t] = \frac{d^2}{\nu} \quad \text{(viscous time)}$$

$$[T] = \Delta T$$

$$[B] = \sqrt{\mu_0 \lambda \tilde{\rho} \Omega}$$

Background state (if any)

$$[\tilde{\rho}] = \tilde{\rho}(r_o) \quad ; \quad [\tilde{T}] = \tilde{T}(r_o)$$



King 2010

- ν = kinematic viscosity
- $\lambda = 1/(\mu_0 \sigma)$ = magnetic diffusivity

MHD equations

Anelastic equations: LBR formulation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\,\mathbf{v} = -\nabla\left(\frac{P'}{\rho_a}\right) + \frac{Ra}{Pr} g(r) S \mathbf{e}_r - \frac{2}{E}\mathbf{e}_z \times \mathbf{v} + \mathbf{F}_\nu + \frac{(\nabla\times\mathbf{B})\times\mathbf{B}}{\rho_a E Pm}$$

$$\rho_a T_a \left[\frac{\partial S}{\partial t} + (\mathbf{v}\cdot\nabla)\,S\right] = \frac{1}{Pr}\nabla\cdot(\rho_a T_a \nabla S) + \frac{Di\,Pr}{Ra}\left(Q_\nu + \frac{1}{Pm^2 E}(\nabla\times\mathbf{B})^2\right)$$

$$\nabla\cdot(\rho_a \mathbf{v}) = 0$$

$$\nabla\cdot\mathbf{B} = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}) + \frac{1}{Pm}\nabla^2\mathbf{B}$$

with $\begin{cases} \mathbf{F}_\nu & \text{viscous force} \\ Q_\nu & \text{viscous heating} \end{cases}$

MHD equations

Boussinesq limit

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla P' + \frac{Ra}{Pr} g(r) S \mathbf{e}_r - \frac{2}{E}\mathbf{e}_z \times \mathbf{v} + \nabla^2 \mathbf{v} + \frac{1}{EPm}(\nabla\times\mathbf{B})\times\mathbf{B}$$

$$\frac{\partial T}{\partial t} + (\mathbf{v}\cdot\nabla)\, T = \frac{1}{Pr}\nabla^2 T$$

$$\nabla\cdot\mathbf{v} = 0$$

$$\nabla\cdot\mathbf{B} = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}) + \frac{1}{Pm}\nabla^2\mathbf{B}$$

Boundary conditions

- Mechanical boundary conditions: no slip or stress-free
- Thermal boundary conditions: fixed temperature or fixed flux
- Magnetic boundary conditions: vacuum, pseudo-vacuum, ...

Control parameters

Rayleigh number $Ra = \dfrac{\alpha_o T_o g_o d^3 \Delta S}{c_p \nu \kappa}$

Ekman number $E = \dfrac{\nu}{\Omega d^2}$

Prandtl number $Pr = \nu/\kappa$

magnetic Prandtl number $Pm = \nu/\eta$

shell aspect ratio $\chi = r_\mathrm{i}/r_\mathrm{o}$

Anelastic with a polytropic reference state, $\tilde{T}$, $\tilde{\rho} = \tilde{T}^n$, $\tilde{P} = \tilde{T}^{n+1}$

density contrast $N_\rho = \ln \dfrac{\tilde{\rho}_i}{\tilde{\rho}_o}$

polytropic index n

Physical parameter regime and numerical limitations

Parameter	Earth's core	Gas giants	Sun	Simulations
E	10^{-15}	10^{-18}	10^{-15}	10^{-6}
Ra	10^{27}	10^{30}	10^{24}	10^{12}
Pr	10^{-1}	10^{-1}	10^{-6}	10^{-1}
Pm	10^{-6}	10^{-7}	10^{-3}	10^{-1}
Re	10^{9}	10^{12}	10^{13}	10^{3}

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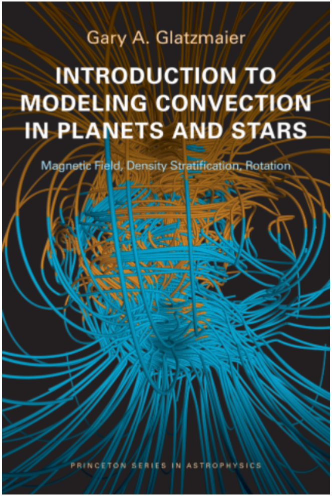
Conclusion

MagIC PROS

- open source, available on Github
- documented DOC
- benchmarked (Boussinesq & anelastic)
- post-processing Python module

Bibliography

- Astrosim 2017 scienceconf.org
- Glatzmaier, Introduction to Modeling Convection in Planets and Stars, 2013



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`input.nml`

Namelist sections

1. Grid parameter
   N_r, N_ϕ, *minc* (rotational symmetry)
2. Control namelist
   (mode selection, iterations, run time (useful on clusters), tag)
3. Physical parameters
   (control parameters, boundary conditions, gravity profile)
4. Initial conditions
5. Ouput parameters

NB: default values are found in Namelist.f90

General structure of the code

1. precalculations
2. time loop:
   - radialLoop : non-linear terms
   - LMLoop : spectral space
3. finalisation (restart file, averaged quantities)

Compiling the code

1. `source magic/sourceme.sh`
2. edit `magic/src/Makefile` and compile with `make -j`
    - gnu compilers
    - internal libraries (JW)
    - serial version

“Real” use

1. SHTNS library
2. MPI/OpenMP

Rem: python module

have in mind the backend in `magic/python/magic/magic.cfg`

Boussinesq convection

1. edit the provided input file to reproduce the Boussinesq benchmark (Christensen et al. 2001)

$$Ra = 10^5 \quad ; \quad E = 10^{-3} \quad ; \quad Pr = 1 \quad ; \quad \chi = 0.35$$

Run the code

1. plot the kinetic energy time series
2. compare the symmetric/anti-symmetric kinetic energies
3. compare it with the benchmark energy density value $\sim$58
4. make an equatorial slice of the velocity field (Surf)
5. try visualisation with paraview (VTK)
6. try to illustrate the structure of the flow (contour, streamlines, shell boundaries)
7. look and comment at the kinetic energy spectra
8. plot the radial kinetic energy profiles

Parameter study ; around the benchmark

Increase the Rayleigh number by roughfly a factor 3, 9, 10^3

1. doing so, restart from the previous simulation
2. compare the kinetic time series, heat transfer
3. plot the average Reynolds number as a function of *Ra* (AvgField)
4. increase the resolution

Lower the Ekman number to 10^{-4}

1. what happens ? comment

Changing the mechanical boundary conditions

1. switch from no-slip to stress-free boundary conditions (Rem: angular momentum correction)
2. compare the axisymmetric toroidal kinetic energies
3. compare the meridional slices(/averages) of the azimuthal velocity field for the different boundary conditions

MHD cases (i)

Starting from the hydrodynamical benchmark, set up an MHD simulation starting with dipolar field (init_b1=3) of amplitude $\Lambda = 5$ at $Pm = 5$. Iterate over 30000 timesteps.

1. explore the sensitivity to the initial conditions : e.g, decrease the initial amplitude by a factor 10.
2. lower *Pm* ; give a bound for the critical *Pm* to get a dynamo
3. what are the default magnetic boundary conditions ? visualize the magnetic field lines outside the computational volume (potential extrapolation)

MHD cases (ii)

Run an hydro simulation at Ra=3e5 (integrate at leat for 30000 iterations).

1. test if this flow is a kinematic dynamo ?
2. restart from this base flow with a weak dipole field ($\Lambda = 0.1$)? compare the system evolution for the full MHD case and the case where you just turn off the induction equation
3. try to make butterfly diagrams (B_r at depth $0.8r_o$). comment
4. is the resolution sufficient ?

Anelastic convection

Run anelastic simulations with a central mass distribution, $n = 2$ polytropic index, $E = 3 \times 10^4$ and increase the density stratification. Resolution N_R = 65, N_ϕ=192, minc=4.

N_ρ	Ra
1	7×10^4
2	2×10^5
4	4.2×10^5

1. show the evolution of the adiabatic background state
2. how does the order of the critical mode change ?